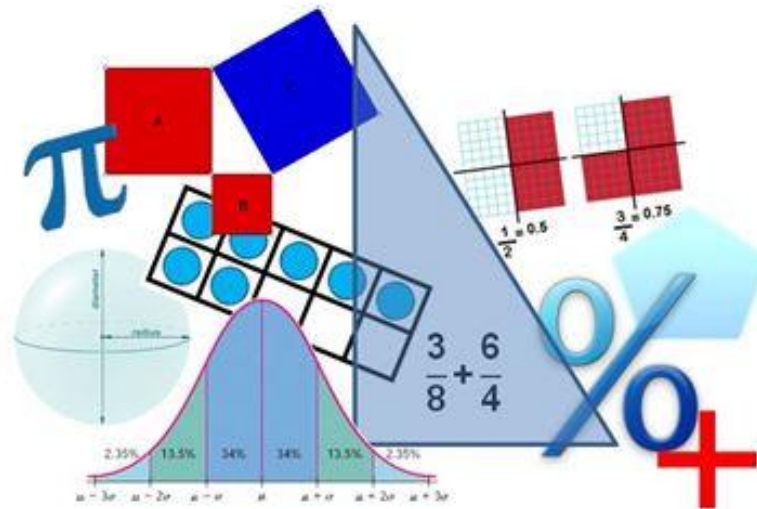


# Mathematics

## 2016 Standards of Learning

### Grade 3

### Curriculum Framework



Board of Education  
Commonwealth of Virginia

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**NOTICE**

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# Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

## Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

### *Understanding the Standard*

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

### *Essential Knowledge and Skills*

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

## **Mathematical Process Goals for Students**

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The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

### **Mathematical Problem Solving**

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

### **Mathematical Communication**

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

### **Mathematical Reasoning**

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

### **Mathematical Connections**

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

### **Mathematical Representations**

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

## **Instructional Technology**

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The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “...the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (\*).

## **Computational Fluency**

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Mathematics instruction must develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

## **Algebra Readiness**

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The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

## Equity

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“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades three through five should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades three through five allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g.,  $\frac{1}{2}$  or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

## 3.1 The student will

- a) read, write, and identify the place and value of each digit in a six-digit whole number, with and without models;
- b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
- c) compare and order whole numbers, each 9,999 or less.

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>• The structure of the base-ten number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This structure, known as a ten-to-one place value relationship, is helpful in comparing and ordering numbers.</li> <li>• Models that clearly illustrate the relationships among hundreds, tens, and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece).</li> <li>• Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the 8 is in the hundreds place, and the value of the 8 is eight hundred.</li> <li>• Flexibility in thinking about numbers — or “decomposition” of numbers (e.g., 2,345 is 23 hundreds, 4 tens, and 5 ones, or 2 thousands, 34 tens, and 5 ones, or 22 hundreds, 13 tens, and 15 ones, etc.) — is critical and supports understandings essential to addition/subtraction and multiplication/division. This flexibility also builds background understanding for the ideas that students use when regrouping (e.g., When subtracting 18 from 174, a student may choose to regroup and think of 174 as 1 hundred, 6 tens, and 14 ones, while another child might regroup 174 as 1 hundred, 5 tens, and 24 ones. Then subtract 18 from 24.).</li> <li>• Whole numbers may be written in a variety of formats:             <ul style="list-style-type: none"> <li>– Standard: 123,456;</li> <li>– Written: one hundred twenty-three thousand, four hundred fifty-six; and</li> <li>– Expanded: <math>100,000 + 20,000 + 3,000 + 400 + 50 + 6</math></li> </ul> </li> <li>• Numbers are arranged into groups of three places called <i>periods</i> (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students determine the value of a digit in any number as well as read and write numbers.</li> <li>• Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students’ environment). Rounding is an estimation strategy that is often used to assess the reasonableness of a solution or to estimate an amount.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Read six-digit numerals orally. (a)</li> <li>• Write six-digit numerals in standard form that are stated verbally or written in words. (a)</li> <li>• Represent numbers up to 9,999 in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, but also 25 tens and 6 ones), with and without models. (a)</li> <li>• Determine the value of each digit in a six-digit whole number (e.g., in 165,724, the 7 represents 7 hundreds and its value is 700). (a)</li> <li>• Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. (b)</li> <li>• Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. (b)</li> <li>• Compare two whole numbers, each 9,999 or less, using symbols (<math>&gt;</math>, <math>&lt;</math>, <math>=</math>, or <math>\neq</math>) and/or words (<i>greater than</i>, <i>less than</i>, <i>equal to</i>, and <i>not equal to</i>). (c)</li> <li>• Order up to three whole numbers, each 9,999 or less, represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c)</li> </ul> |

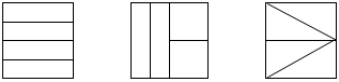
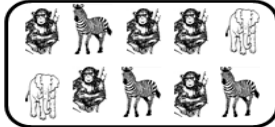


- 3.1 The student will**
- a) read, write, and identify the place and value of each digit in a six-digit whole number, with and without models;**
  - b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and**
  - c) compare and order whole numbers, each 9,999 or less.**

| Understanding the Standard   | Essential Knowledge and Skills |
|--|--------------------------------|
| <ul style="list-style-type: none"><li>• Students should explore reasons for estimation, using practical experiences, and use rounding to solve practical problems.</li><li>• The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine the closest multiples of ten, hundred, or thousand it is between. Then, identify to which it is closer.</li></ul> |                                |

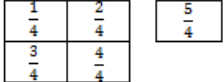
## 3.2 The student will

- name and write fractions and mixed numbers represented by a model;
- represent fractions and mixed numbers, with models and symbols; and
- compare fractions having like and unlike denominators, using words and symbols ( $>$ ,  $<$ ,  $=$ , or  $\neq$ ), with models.

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>When working with fractions, the whole must be defined.</li> <li>A fraction is a numerical way of representing part of a whole region (i.e., an area model), part of a group (as in a set model), or part of a length (i.e., a measurement model).</li> <li>Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator (e.g., <math>\frac{3}{2}</math>). An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., <math>1\frac{1}{2}</math>).</li> <li>The value of a fraction is dependent on both the number of equivalent parts in a whole (denominator) and the number of those parts being considered (numerator).</li> <li>A fractional part of a whole can be modeled using             <ul style="list-style-type: none"> <li>region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings);</li> <li>set models (e.g., chips, counters, cubes, drawings); and</li> <li>length/measurement models (e.g., rods, connecting cubes, number lines, rulers, and drawings).</li> </ul> </li> <li>In each area/region model, the whole is divided or partitioned into parts with area of equivalent value. The fractional parts are not always congruent and could have a different shape as shown in the examples:             <div style="text-align: center;">  </div> </li> <li>In the set model, each member of the set is an equivalent part of the set. In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance, if a whole is defined as a set of 10 animals, the animals within the set may be different. For example, students should be able to identify apes as representing half of the animals in the set shown:             <div style="text-align: center;">  </div> </li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Name and write fractions (proper and improper) and mixed numbers with denominators of 12 or less in symbols represented by concrete and/or pictorial models. (a)</li> <li>Represent a given fraction (proper or improper) and mixed numbers, using concrete or pictorial set, area/region, length/measurement models and symbols. (b)</li> <li>Identify a fraction represented by a model as the sum of unit fractions. (b)</li> <li>Using a model of a fraction greater than one, count the fractional parts to name and write it as an improper fraction and as a mixed number (e.g., <math>\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4} = 1\frac{1}{4}</math>, or <math>2\frac{1}{3} = \frac{7}{3}</math>). (b)</li> <li>Compare a model of a fraction, less than or equal to one, to the benchmarks of <math>0, \frac{1}{2}</math>, and <math>1</math>. (c)</li> <li>Compare proper fractions using the terms <i>greater than</i>, <i>less than</i>, <i>equal to</i>, or <i>not equal to</i> and the symbols (<math>&lt;</math>, <math>&gt;</math>, <math>=</math>, and <math>\neq</math>). Comparisons are made between fractions with both like and unlike denominators, with concrete or pictorial models. (c)</li> </ul> |

## 3.2 The student will

- a) name and write fractions and mixed numbers represented by a model;
- b) represent fractions and mixed numbers, with models and symbols; and
- c) compare fractions having like and unlike denominators, using words and symbols ( $>$ ,  $<$ ,  $=$ , or  $\neq$ ), with models.

| Understanding the Standard   | Essential Knowledge and Skills |
|--|--------------------------------|
| <ul style="list-style-type: none"> <li>• In the primary grades, students may benefit from experiences with sets that are comprised of congruent figures (e.g., 12 eggs in a carton) before working with sets that have noncongruent parts.</li> <li>• Students need opportunities to use models to count fractional parts that go beyond a whole. For instance, if students are counting five slices of cake and building the cake as they count, where each slice is equivalent to one-fourth, they might say “one-fourth, two-fourths, three-fourths, four-fourths, five-fourths.” As a result of building the whole while they are counting, they begin to realize that four-fourths make one whole and the fifth-fourth starts another whole, and they begin to develop flexibility in naming this amount in different ways (e.g., five-fourths or one and one-fourth). They will begin to generalize that when the numerator and the denominator are the same, there is one whole and when the numerator is larger than the denominator, there is more than one whole. They also will begin to see a fraction as the sum of unit fractions (e.g., three-fourths contains three one-fourths or four-fourths contains four one-fourths which is equal to one whole). This provides students with a visual, as in the example below, for when one whole is reached and develops a greater understanding of numerator and denominator.</li> </ul> <div style="text-align: center;">  <math display="block">\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \text{ and } \frac{5}{4} = 1\frac{1}{4}</math> </div> <ul style="list-style-type: none"> <li>• Models, benchmarks, and equivalent forms are helpful in judging the size of fractions.</li> <li>• Experiences at this level should include exploring and reasoning about comparing and ordering fractions with common numerators, common denominators, or comparing to the benchmarks of <math>\frac{1}{2}</math> and 1.</li> <li>• Students should have a variety of experiences focusing on comparing: <ul style="list-style-type: none"> <li>– fractions with like denominators;</li> <li>– fractions with like numerators;</li> <li>– fractions that are more than one whole and less than one whole; and</li> <li>– fractions close to zero, close to one-half, and close to one whole.</li> </ul> </li> </ul> |                                |

## 3.2 The student will

- a) name and write fractions and mixed numbers represented by a model;
- b) represent fractions and mixed numbers, with models and symbols; and
- c) compare fractions having like and unlike denominators, using words and symbols ( $>$ ,  $<$ ,  $=$ , or  $\neq$ ), with models.

| Understanding the Standard  | Essential Knowledge and Skills |
|---|--------------------------------|
| <ul style="list-style-type: none"> <li>• Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., <math>\frac{1}{5}</math> of a bar is smaller than <math>\frac{1}{4}</math> of the same bar).</li> <li>• Comparing fractions to a benchmark on a number line (e.g., close to 0, less than <math>\frac{1}{2}</math>, exactly <math>\frac{1}{2}</math>, greater than <math>\frac{1}{2}</math>, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.</li> <li>• Provide opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations.</li> <li>• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., <i>thirds</i> means “three equal parts of a whole,” <math>\frac{1}{3}</math> represents one of three equal-size parts when a pizza is shared among three students, or <i>three-fourths</i> means “three of four equal parts of a whole”) furthers this development.</li> </ul> |                                |

Computation and estimation in grades three through five should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade five.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships between operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems (e.g.,  $7 \times 28$  is equivalent to  $(7 \times 20) + (7 \times 8)$ ).

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals using benchmarks (e.g.,  $\frac{2}{5} + \frac{1}{3}$  must be less than one because both fractions are less than  $\frac{1}{2}$ ). Using estimation, students should develop strategies to recognize the reasonableness of their solutions.

Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

- 3.3 The student will**
- a) estimate and determine the sum or difference of two whole numbers; and**
  - b) create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.**

| Understanding the Standard   | Essential Knowledge and Skills   |
|--|--|
| <ul style="list-style-type: none"> <li>• Flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values, are useful.</li> <li>• Grade three students should explore and apply the properties of addition as strategies for solving addition and subtraction problems using a variety of representations (e.g., manipulatives, diagrams, symbols, etc.).</li> <li>• The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties of addition are most appropriate for exploration at this level:             <ul style="list-style-type: none"> <li>– The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., <math>4 + 3 = 3 + 4</math>).</li> <li>– The identity property of addition states that if zero is added to a given number, the sum is the same as the given number.</li> <li>– The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., <math>15 + (35 + 16) = (15 + 35) + 16</math>).</li> </ul> </li> <li>• Using concrete materials (e.g., base-ten blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations which helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms.</li> <li>• Exploring concepts through concrete experiences develops conceptual understanding. Next, the children must make connections that serve as a bridge to the symbolic. Student-created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections.</li> <li>• Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problems involving single-step and multistep problems. (a, b)</li> <li>• Estimate the sum of two whole numbers with sums to 9,999. (a)</li> <li>• Estimate the difference of two whole numbers, each 9,999 or less. (a)</li> <li>• Apply strategies, including place value and the properties of addition, to add two whole numbers with sums to 9,999. (a, b)</li> <li>• Apply strategies, including place value and the properties of addition, to subtract two whole numbers, each 9,999 or less. (a, b)</li> <li>• Use inverse relationships between addition and subtraction facts to solve practical problems. (b)</li> <li>• Create and solve single-step and multistep practical problems involving the sum or difference of two whole numbers, each 9,999 or less. (b)</li> </ul> |

3.3 The student will

- a) estimate and determine the sum or difference of two whole numbers; and
- b) create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.

| Understanding the Standard  | Essential Knowledge and Skills  |   |  |
|---|---|---|--|
| <ul style="list-style-type: none"> <li>• Students should experience a variety of problem types related to addition and subtraction. Examples are included in the following chart:</li> </ul>                            |   |   |  |
| <b>GRADE 3: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES</b>   |   |   |  |
| Join<br>(Result Unknown)  | Join<br>(Change Unknown)  | Join<br>(Start Unknown)   |  |
| Sue had 214 sheets of paper. Alex gave her 95 more sheets of paper. How many sheets of paper does Sue have altogether?  | Sue had 214 sheets of paper. Alex gave her some more sheets of paper. Now Sue has 309 sheets of paper. How many did Alex give her?  | Sue had some sheets of paper. Alex gave her 95 more sheets of paper. Now Sue has 309 sheets of paper. How many sheets of paper did Sue have to start with?  |  |
| Separate<br>(Result Unknown)  | Separate<br>(Change Unknown)  | Separate<br>(Start Unknown)   |  |
| Brooke had 145 marbles. She gave 26 marbles to Joe. How many marbles does Brooke have now?  | Brooke had 145 marbles. She gave some to Joe. She has 119 marbles left. How many marbles did Brooke give to Joe?  | Brooke had some marbles. She gave 26 to Joe. Now she has 119 marbles left. How many marbles did Brooke start with?  |  |
| Part-Part-Whole<br>(Whole Unknown)  | Part-Part-Whole<br>(One Part Unknown)   | Part-Part-Whole<br>(Both Parts Unknown)   |  |
| There are 29 boys and 36 girls in the third grade at a school. How many boys and girls are in third grade in all?   | There are 65 students in third grade. Twenty-nine of the students are boys, and the rest are girls. How many girls are in third grade?  | There are 65 students in third grade. Some of the students are girls and some of the students are boys. How many students could be girls and how many students could be boys?   |  |
| Compare<br>(Difference Unknown)   | Compare<br>(Bigger Unknown)   | Compare<br>(Smaller Unknown)  |  |
| Mr. Ross has 325 books. Mrs. King has 196 books. How many more books does Mr. Ross have than Mrs. King?<br><br>Mr. Ross has 325 books. Mrs. King has 196 books. How many fewer books does Mrs. King have than Mr. Ross? | Mrs. King has 196 books. Mr. Ross has 129 more books than Mrs. King. How many books does Mr. Ross have?<br><br>Mrs. King has 129 fewer books than Mr. Ross. Mrs. King has 196 books. How many books does Mr. Ross have? | Mr. Ross has 129 more books than Mrs. King. Mr. Ross has 325 books. How many books does Mrs. King have?<br><br>Mrs. King has 129 fewer books than Mr. Ross. Mr. Ross has 325 books. How many books does Mrs. King have? |  |

## 3.3 The student will

- a) estimate and determine the sum or difference of two whole numbers; and
- b) create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.

| Understanding the Standard  | Essential Knowledge and Skills |
|---|--------------------------------|
| <ul style="list-style-type: none"> <li>• Combination problem types are introduced in grade five (e.g., How many different outfits can be made given 3 shirts and two pants?).</li> <li>• In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as <i>in all</i>, <i>altogether</i>, <i>difference</i>, etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.</li> <li>• Addition is the combining of quantities; it uses the following terms:           <ul style="list-style-type: none"> <li><math>addend \rightarrow 423</math></li> <li><math>addend \rightarrow + 246</math></li> <li><math>sum \rightarrow 669</math></li> </ul> </li> <li>• Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms:           <ul style="list-style-type: none"> <li><math>minuend \rightarrow 7,698</math></li> <li><math>subtrahend \rightarrow -5,341</math></li> <li><math>difference \rightarrow 2,357</math></li> </ul> </li> <li>• An algorithm is a step-by-step method for computing.</li> <li>• The least number of steps necessary to solve a single-step problem is one.</li> <li>• Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed.</li> <li>• Estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer.</li> <li>• When an exact answer is required, opportunities to explore whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the most efficient approach.</li> </ul> |                                |


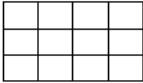
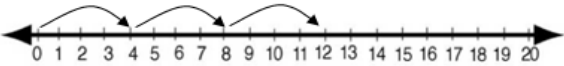


- 3.3 The student will**
- a) estimate and determine the sum or difference of two whole numbers; and**
  - b) create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.**

| Understanding the Standard  | Essential Knowledge and Skills |
|---|--------------------------------|
| <ul style="list-style-type: none"> <li>• Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation.</li> <li>• There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include:               <ul style="list-style-type: none"> <li>– adding doubles;</li> <li>– using addition by counting up for solving subtraction problems;</li> <li>– adding 9: add 10 and subtract 1; and</li> <li>– making 10: for column addition, looking for numbers that can be grouped together to make 10.</li> </ul> </li> </ul> |                                |

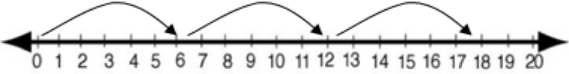
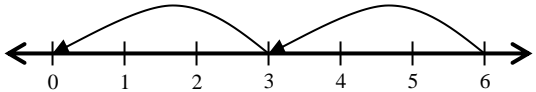
## 3.4 The student will

- a) represent multiplication and division through  $10 \times 10$ , using a variety of approaches and models;
- b) create and solve single-step practical problems that involve multiplication and division through  $10 \times 10$ ;
- c) demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and
- d) solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.

| Understanding the Standard  | Essential Knowledge and Skills  |
|---|---|
| <ul style="list-style-type: none"> <li>• Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and practical problems involving equal-sized groups, arrays, and length models.</li> <li>• To extend the understanding of multiplication, three models may be used:               <ul style="list-style-type: none"> <li>– The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces the concept of multiplication as a way to find the total number of items in a collection of groups, with the same amount in each group, and the total number of items can be found by repeated addition or skip counting.                   <div style="text-align: center; margin: 10px 0;">  </div> </li> <li>– The array model, consisting of rows and columns (e.g., three rows of four columns for a 3-by-4 array), helps build an understanding of the commutative property.                   <div style="text-align: center; margin: 10px 0;">  </div> </li> <li>– The length model (e.g., a number line) also reinforces repeated addition or skip counting.                   <div style="text-align: center; margin: 10px 0;">  </div> </li> </ul> </li> <li>• The terms associated with multiplication are listed below:               <ul style="list-style-type: none"> <li><i>factor</i> → 54</li> <li><i>factor</i> → <math>\times 3</math></li> <li><i>product</i> → 162</li> </ul> </li> <li>• There is an inverse relationship between multiplication and division.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication using a variety of approaches and models (e.g., repeated addition, equal-sized groups, arrays, equal jumps on a number line, skip counting). (a)</li> <li>• Represent division using a variety of approaches and models (e.g., repeated subtraction, equal sharing, equal groups). (a)</li> <li>• Write three related equations (fact sentences) when given one equation (fact sentence) for multiplication or division (e.g., given <math>6 \times 7 = 42</math>, write <math>7 \times 6 = 42</math>, <math>42 \div 7 = 6</math>, and <math>42 \div 6 = 7</math>). (a)</li> <li>• Create practical problems to represent a multiplication or division fact. (b)</li> <li>• Use multiplication and division basic facts to represent a given situation, using a number sentence. (b)</li> <li>• Recognize and use the inverse relationship between multiplication and division to solve practical problems. (b)</li> <li>• Solve single-step practical problems that involve multiplication and division of whole numbers through <math>10 \times 10</math>. (b)</li> <li>• Demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10. (c)</li> <li>• Solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. (d)</li> </ul> |

## 3.4 The student will

- represent multiplication and division through  $10 \times 10$ , using a variety of approaches and models;
- create and solve single-step practical problems that involve multiplication and division through  $10 \times 10$ ;
- demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and
- solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>The number line model can be used to solve a multiplication problem such as <math>3 \times 6</math>. This is represented on the number line by three jumps of six or six jumps of three, depending on the context of the problem.            </li> <li>The number line model can also be used to solve a division problem such as <math>6 \div 3</math> and is represented on the number line by noting how many jumps of three go from six to zero.            <p>The number line model above shows two jumps of three between 6 and 0, answering the question of how many jumps of three go from 6 to 0; therefore, <math>6 \div 3 = 2</math>.</p> </li> <li>Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.</li> <li>The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts. By studying patterns and relationships, students build a foundation for fluency with multiplication and division facts.</li> <li>Beginning with learning the foundational multiplication facts for 0, 1, 2, 5, and 10 allows students to utilize prior skip counting skills and the use of doubles to solve problems. Understanding and using the foundational facts can be helpful in deriving and learning all multiplication facts. For example, decomposing one of the factors in <math>7 \times 6</math>, allows for the use of the foundational facts of 5s and 2s. This knowledge can be combined to learn the facts for 7 (e.g., <math>7 \times 6</math> can be thought of as <math>(5 \times 6) + (2 \times 6)</math>).</li> </ul> | <ul style="list-style-type: none"> <li>Apply strategies, including place value and the properties of multiplication and/or addition when multiplying and dividing whole numbers. (a, b, c, d)</li> </ul> |

## 3.4 The student will

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| Understanding the Standard  | Essential Knowledge and Skills |
|---|--------------------------------|
| <ul style="list-style-type: none"> <li>• As students work to solve multiplication and division problems, they naturally tend to utilize strategies that involve place value understanding and properties of the operations. Applying the commutative property of multiplication (e.g., <math>5 \times 8 = 8 \times 5</math>) reduces in half the number of multiplication facts that students must learn. The distributive property of multiplication allows students to find the answer to a problem such as <math>6 \times 7</math> by decomposing 7 into 3 and 4 (e.g., <math>6 \times 7 = 6 \times (3 + 4)</math>) allowing them to think about <math>(6 \times 3) + (6 \times 4) = 18 + 24 = 42</math>.</li> <li>• Strategies that allow students to derive unknown facts from facts they do know include: doubles (2s facts), doubling twice (4s facts), five facts (half of ten), decomposing into known facts (e.g., <math>7 \times 8</math> can be thought of as <math>(5 \times 8) + (2 \times 8)</math>).</li> <li>• Strategies for solving problems that involve multiplication or division may include mental strategies, partial products, the standard algorithm, and the commutative, associative, and distributive properties.</li> <li>• An algorithm is a step-by-step method for computing.</li> <li>• The least number of steps necessary to solve a single-step problem is one.</li> <li>• Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types.</li> <li>• Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:</li> </ul> |                                |

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| Understanding the Standard  | Essential Knowledge and Skills   |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
|---|--|---|--|----------------------|--|--|-----------------------------------|--|--|---|--|---|------------------------------------|--|--|----------------|---------------|---------------------------|--|---|--|----------------|--|--|---------------|-----------------------|--|--|--|--|--|---|--|---|--|--|--|
| <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr style="background-color: #d3d3d3;"> <th colspan="3">GRADE 3: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES</th> </tr> <tr style="background-color: #d3d3d3;"> <th colspan="3">Equal Group Problems</th> </tr> <tr> <th style="width: 33%;">Whole Unknown<br/>(Multiplication)</th> <th style="width: 33%;">Size of Groups Unknown<br/>(Partitive Division)</th> <th style="width: 33%;">Number of Groups Unknown<br/>(Measurement Division)</th> </tr> </thead> <tbody> <tr> <td style="text-align: left; padding: 2px;">There are 5 boxes of markers. Each box contains 6 markers. How many markers are there in all?</td> <td style="text-align: left; padding: 2px;">If 30 markers are shared equally among 5 friends, how many markers will each friend get?</td> <td style="text-align: left; padding: 2px;">If 30 markers are placed into school boxes with each box containing 6 markers, how many school boxes can be filled?</td> </tr> <tr style="background-color: #d3d3d3;"> <th colspan="3">Multiplicative Comparison Problems</th> </tr> <tr> <th>Result Unknown</th> <th>Start Unknown</th> <th>Comparison Factor Unknown</th> </tr> <tr> <td style="text-align: left; padding: 2px;">Tyrone ran 3 miles. Jasmine ran 4 times as many miles as Tyrone. How many miles did Jasmine run?</td> <td style="text-align: left; padding: 2px;">Jasmine ran 12 miles. She ran 4 times as many miles as Tyrone. How many miles did Tyrone run?</td> <td style="text-align: left; padding: 2px;">Jasmine ran 12 miles. Tyrone ran 3 miles. How many times more miles did Jasmine run than Tyrone?</td> </tr> <tr style="background-color: #d3d3d3;"> <th colspan="3">Array Problems</th> </tr> <tr> <th>Whole Unknown</th> <th colspan="2">One Dimension Unknown</th> </tr> <tr> <td style="text-align: left; padding: 2px;">There were 3 baseball teams competing at the field. Each team had 9 baseball players. How many baseball players were there all together?</td> <td colspan="2" style="text-align: left; padding: 2px;">There are 27 children playing on teams at the field. The children are divided equally among 3 teams. How many children are on each team?</td> </tr> <tr> <td></td> <td colspan="2" style="text-align: left; padding: 2px;">There are 27 children playing on teams at the field. There are 9 children on each team. How many teams are there?</td> </tr> <tr> <td colspan="3" style="padding: 2px;">Note: Area problems will be included in Grades 4 and 5.</td> </tr> </tbody> </table> | GRADE 3: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES  |   |  | Equal Group Problems |  |  | Whole Unknown<br>(Multiplication) | Size of Groups Unknown<br>(Partitive Division) | Number of Groups Unknown<br>(Measurement Division) | There are 5 boxes of markers. Each box contains 6 markers. How many markers are there in all? | If 30 markers are shared equally among 5 friends, how many markers will each friend get? | If 30 markers are placed into school boxes with each box containing 6 markers, how many school boxes can be filled? | Multiplicative Comparison Problems |  |  | Result Unknown | Start Unknown | Comparison Factor Unknown | Tyrone ran 3 miles. Jasmine ran 4 times as many miles as Tyrone. How many miles did Jasmine run? | Jasmine ran 12 miles. She ran 4 times as many miles as Tyrone. How many miles did Tyrone run? | Jasmine ran 12 miles. Tyrone ran 3 miles. How many times more miles did Jasmine run than Tyrone? | Array Problems |  |  | Whole Unknown | One Dimension Unknown |  | There were 3 baseball teams competing at the field. Each team had 9 baseball players. How many baseball players were there all together? | There are 27 children playing on teams at the field. The children are divided equally among 3 teams. How many children are on each team? |  |  | There are 27 children playing on teams at the field. There are 9 children on each team. How many teams are there? |  | Note: Area problems will be included in Grades 4 and 5. |  |  |  |
| GRADE 3: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES   |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| Equal Group Problems  |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| Whole Unknown<br>(Multiplication)   | Size of Groups Unknown<br>(Partitive Division)   | Number of Groups Unknown<br>(Measurement Division)  |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
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| Multiplicative Comparison Problems  |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| Result Unknown  | Start Unknown  | Comparison Factor Unknown   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
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| Array Problems  |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| Whole Unknown   | One Dimension Unknown  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| There were 3 baseball teams competing at the field. Each team had 9 baseball players. How many baseball players were there all together?  | There are 27 children playing on teams at the field. The children are divided equally among 3 teams. How many children are on each team? |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
|   | There are 27 children playing on teams at the field. There are 9 children on each team. How many teams are there?                        |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| Note: Area problems will be included in Grades 4 and 5.   |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |
| <ul style="list-style-type: none"> <li>• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers.</li> </ul>  |  |   |  |                      |  |  |                                   |  |  |   |  |   |                                    |  |  |                |               |                           |  |   |  |                |  |  |               |                       |  |  |  |  |  |   |  |   |  |  |  |

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| Understanding the Standard   | Essential Knowledge and Skills |                         |                               |                                |           |            |      |       |  |
|--|--------------------------------|-------------------------|-------------------------------|--------------------------------|-----------|------------|------|-------|--|
| <ul style="list-style-type: none"> <li>• Grade three students should explore and apply the properties of multiplication and addition as strategies for solving multiplication and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols).</li> <li>• The properties of the operations are “rules” about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level:               <ul style="list-style-type: none"> <li>– The commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., <math>2 \times 3 = 3 \times 2</math>).</li> <li>– The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number.</li> <li>– The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., <math>15 + (35 + 16) = (15 + 35) + 16</math>).</li> <li>– The associative property of multiplication states that the product stays the same when the grouping of factors is changed (e.g., <math>6 \times (3 \times 5) = (6 \times 3) \times 5</math>).</li> <li>– The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products:                   <table style="margin-left: 40px; border: none;"> <tr> <td style="padding-right: 20px;"><math>8 \times 7 = 8(5 + 2)</math></td> <td><math>5 \times 23 = 5(20+3)</math></td> </tr> <tr> <td><math>(8 \times 5) + (8 \times 2)</math></td> <td><math>(5 \times 20) + (5 \times 3)</math></td> </tr> <tr> <td><math>40 + 16</math></td> <td><math>100 + 15</math></td> </tr> <tr> <td><math>56</math></td> <td><math>115</math></td> </tr> </table> </li> </ul> </li> </ul> | $8 \times 7 = 8(5 + 2)$        | $5 \times 23 = 5(20+3)$ | $(8 \times 5) + (8 \times 2)$ | $(5 \times 20) + (5 \times 3)$ | $40 + 16$ | $100 + 15$ | $56$ | $115$ |  |
| $8 \times 7 = 8(5 + 2)$  | $5 \times 23 = 5(20+3)$        |                         |                               |                                |           |            |      |       |  |
| $(8 \times 5) + (8 \times 2)$  | $(5 \times 20) + (5 \times 3)$ |                         |                               |                                |           |            |      |       |  |
| $40 + 16$  | $100 + 15$                     |                         |                               |                                |           |            |      |       |  |
| $56$   | $115$                          |                         |                               |                                |           |            |      |       |  |

- 3.5 The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less.**

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>• Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., <math>3\frac{5}{8}</math>).</li> <li>• When adding or subtracting fractions, an answer greater than one can be expressed as an improper fraction or the equivalent mixed number (e.g., <math>\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}</math>).</li> <li>• When adding and subtracting fractions the fractions must represent like size units (e.g., one-fifth added to three-fifths is four-fifths). This understanding builds the foundation for why common denominators are necessary in future work with adding unlike fractions and for work in algebra when adding polynomial expressions.</li> <li>• Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as <math>0</math>, <math>\frac{1}{2}</math>, and <math>1</math>. For example, <math>\frac{3}{5}</math> and <math>\frac{4}{5}</math> are each greater than <math>\frac{1}{2}</math>, so their sum is greater than <math>1</math>.</li> <li>• Concrete materials and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 12 or less.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less, using concrete and pictorial models representing area/regions (e.g., circles, squares, and rectangles), length/measurements (e.g., fraction bars and strips), and sets (e.g., counters).</li> </ul> |

Students in grades three through five should be actively involved in measurement activities that require a dynamic interaction among students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume, area, perimeter, temperature, and time. Student understanding of measurement continues to be enhanced by using appropriate tools such as rulers, balances, clocks, and thermometers.

The study of geometry helps students represent and make sense of the world. In grades three through five, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, draw representations of, and describe the relationships among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence; parallel, intersecting, and perpendicular lines; and classification of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.



- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

- 3.6 The student will**
- a) determine the value of a collection of bills and coins whose total value is \$5.00 or less;**
  - b) compare the value of two sets of coins or two sets of coins and bills; and**
  - c) make change from \$5.00 or less.**

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>• Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is \$5.00 or less.</li> <li>• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills.</li> <li>• A variety of skills can be used to determine the change after a purchase, including:               <ul style="list-style-type: none"> <li>– counting on, using coins and bills (e.g., starting with the amount to be paid (purchase price));</li> <li>– counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and</li> <li>– mentally calculating the difference.</li> </ul> </li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Determine the value of a collection of coins and bills whose total value is \$5.00 or less. (a)</li> <li>• Compare the values of two sets of coins or two sets of coins and bills, up to \$5.00, using the terms <i>greater than</i>, <i>less than</i>, and <i>equal to</i>. (b)</li> <li>• Make change from \$5.00 or less. (c)</li> </ul> |

- 3.7 The student will estimate and use U.S. Customary and metric units to measure**
- length to the nearest  $\frac{1}{2}$  inch, inch, foot, yard, centimeter, and meter; and**
  - liquid volume in cups, pints, quarts, gallons, and liters.**

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing.</li> <li>Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the liquid volume of a small glass of orange juice is about one cup). Practical experiences measuring the volume of familiar objects help to establish benchmarks and facilitate the student's ability to estimate measures.</li> <li>One unit of measure may be more appropriate than another to use when measuring an object, depending on the size of the object and the degree of accuracy desired.</li> <li>Correct use of measurement tools is essential to understanding the concepts of measurement.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest <math>\frac{1}{2}</math> inch, inch, foot, yard, centimeter, and meter. (a)</li> <li>Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest <math>\frac{1}{2}</math> inch, foot, yard, centimeter, and meter. (a)</li> <li>Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. (b)</li> <li>Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. (b)</li> </ul> |

- 3.8 The student will estimate and**
- a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units; and**
  - b) count the number of square units needed to cover a given surface in order to determine its area.**

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>• A polygon is a closed plane figure composed of at least three line segments that do not cross. A plane figure is any closed, two-dimensional shape.</li> <li>• Perimeter is the path or distance around any plane figure.</li> <li>• Area is the number of iterations of a two-dimensional unit needed to cover a surface. The two-dimensional unit is usually a square, but it could also be another shape such as a rectangle or an equilateral triangle.</li> <li>• The unit of measure used to find the perimeter or area is stated along with the numerical value when expressing the perimeter or area of a figure (e.g., the perimeter of the book cover is 38 inches and the area of the book cover is 90 square inches).</li> <li>• Opportunities to explore the concepts of perimeter and area should involve hands-on experiences (e.g., placing toothpicks (units) around a polygon and counting the number of toothpicks to determine its perimeter and filling or covering a polygon with tiles (square units) and counting the tiles to determine its area).</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Estimate and use U.S. Customary and metric units to measure the distance around a polygon with no more than six sides to determine the perimeter. (a)</li> <li>• Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface. (b)</li> </ul> |

- 3.9 The student will**
- tell time to the nearest minute, using analog and digital clocks;
  - solve practical problems related to elapsed time in one-hour increments within a 12-hour period; and
  - identify equivalent periods of time and solve practical problems related to equivalent periods of time.

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>Students need to understand that time has passed or will pass in equal increments (i.e., seconds, minutes, or hours).</li> <li>The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.</li> <li>Students need to understand that there are sixty minutes in an hour when using analog and digital clocks.</li> <li>Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read.</li> <li>Elapsed time is the amount of time that has passed between two given times.</li> <li>Elapsed time should be modeled and demonstrated using geared analog clocks and timelines.</li> <li>Elapsed time can be found by counting on from the beginning time or counting back from the ending time.</li> <li>The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. Students need to know the relationship between periods of time (e.g., If there are 24 hours in one day, how many hours are in three days?).</li> <li>The knowledge that a year has about <math>365\frac{1}{4}</math> days will help students understand the necessity of adding a full day every fourth year, called a <i>leap year</i>.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Tell time to the nearest minute, using analog and digital clocks. (a)</li> <li>Match a written time (e.g., 4:38, 7:09, 12:51) to the time shown on analog and digital clocks to the nearest minute. (a)</li> <li>Solve practical problems related to elapsed time in one-hour increments, within a 12-hour period (within a.m. or within p.m.):             <ul style="list-style-type: none"> <li>when given the beginning time and the ending time, determine the time that has elapsed; (b)</li> <li>when given the beginning time and amount of elapsed time in one-hour increments, determine the ending time; or (b)</li> <li>when given the ending time and the elapsed time in one-hour increments, determine the beginning time. (b)</li> </ul> </li> <li>Identify the number of minutes in an hour and the number of hours in a day. (c)</li> <li>Identify equivalent relationships observed in a calendar, including the approximate number of days in a given month (about 30), the number of days in a week, the number of days in a year (about <math>365\frac{1}{4}</math>), and the number of months in a year. (c)</li> <li>Solve practical problems related to equivalent periods of time to include:             <ul style="list-style-type: none"> <li>approximate days in five or fewer months;</li> <li>days in five or fewer weeks;</li> <li>months in five or fewer years;</li> <li>minutes in five or fewer hours; and</li> <li>hours in five or fewer days. (c)</li> </ul> </li> </ul> |

**3.10 The student will read temperature to the nearest degree.**

| Understanding the Standard  | Essential Knowledge and Skills   |
|---|--|
| <ul style="list-style-type: none"> <li>• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers.</li> <li>• A variety of physical models (e.g., circular and linear) should be used to represent the temperature determined by a real thermometer.</li> <li>• The symbols for degrees in Celsius (<math>^{\circ}\text{C}</math>) and degrees in Fahrenheit (<math>^{\circ}\text{F}</math>) should be used to write temperatures.</li> <li>• Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring things found in the student’s environment (e.g., the temperature of the classroom; temperature on the playground; temperature of warm and cold liquids; body temperature).</li> <li>• At this level, scale increments should be limited to one or two.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Read Celsius and Fahrenheit temperatures to the nearest degree using real thermometers, physical models, or pictorial representations.</li> </ul> |

**3.11 The student will identify and draw representations of points, lines, line segments, rays, and angles.**

| Understanding the Standard  | Essential Knowledge and Skills  |
|---|---|
| <ul style="list-style-type: none"> <li>• A point is an exact location in space. It has no length, width, or height.</li> <li>• A line is a collection of points extending indefinitely in both directions. It has no endpoints.</li> <li>• A line segment is part of a line. It has two endpoints and includes all the points between and including those endpoints.</li> <li>• A ray is part of a line. It has one endpoint and extends indefinitely in one direction.</li> <li>• An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect.</li> <li>• Geometric notation to name lines, line segments and rays is included in grade four.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify examples of points, lines, line segments, rays, and angles.</li> <li>• Describe endpoints and vertices as they relate to lines, line segments, rays, and angles.</li> <li>• Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.</li> </ul> |

## 3.12 The student will

- a) define polygon;
- b) identify and name polygons with 10 or fewer sides; and
- c) combine and subdivide polygons with three or four sides and name the resulting polygon(s).

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>• A polygon is a closed plane figure composed of at least three line segments that do not cross.</li> <li>• Polygons may be described by their attributes (e.g., sides and vertices). Line segments form the sides of a polygon and angles are formed by two line segments coming together at a vertex of a polygon.</li> <li>• A rectangle, square, trapezoid, parallelogram, and rhombus are all classified as quadrilaterals.</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</b></p> <ul style="list-style-type: none"> <li>• Define polygon. (a)</li> <li>• Classify figures as polygons or not polygons. (a)</li> <li>• Identify and name polygons with 10 or fewer sides in various orientations: <ul style="list-style-type: none"> <li>– triangle is a three-sided polygon;</li> <li>– quadrilateral is a four-sided polygon;</li> <li>– pentagon is a five-sided polygon;</li> <li>– hexagon is a six-sided polygon;</li> <li>– heptagon is a seven-sided polygon;</li> <li>– octagon is an eight-sided polygon;</li> <li>– nonagon is a nine-sided polygon; and</li> <li>– decagon is a ten-sided polygon. (b)</li> </ul> </li> <li>• Combine no more than three polygons, where each has three or four sides, and name the resulting polygon. (c)</li> <li>• Subdivide a three-sided or four-sided polygon into no more than three parts and name the resulting polygon(s). (c)</li> </ul> |



**3.13 The student will identify and describe congruent and noncongruent figures.**

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"><li>• Congruent figures have the same size and shape. Noncongruent figures do not have exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models.</li><li>• Congruent plane figures remain congruent even if they are in different spatial orientations.</li><li>• Figures that are congruent or noncongruent may be identified by using direct comparisons and/or tracing procedures.</li></ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"><li>• Identify examples of congruent and noncongruent figures.</li><li>• Determine and explain why plane figures are congruent or noncongruent.</li></ul> |

Students entering grades three through five have begun to explore the concept of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction at grades three through five is to deepen their understanding of the concepts of probability by:

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:


- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

- 3.14 The student will investigate and describe the concept of probability as a measurement of chance and list possible outcomes for a single event.**

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives.</li> <li>• Investigation of experimental probability is continued at this level through informal activities using materials such as two-colored counters, spinners, and random number cubes.</li> <li>• Probability is the measurement of chance of an event occurring.</li> <li>• When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).</li> <li>• Students should have opportunities to describe in informal terms (e.g., <i>impossible, unlikely, equally likely, likely, and certain</i>) the degree of likelihood of an event occurring. Activities should include real-life examples.</li> <li>• For any event, such as flipping a coin, spinning a spinner, or rolling a number cube, the things that can happen are called <i>outcomes</i>. For example, there are two possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up; when flipping a coin, each of the outcomes is equally likely.</li> <li>• All possible outcomes of an experiment may be organized in a list, table, or chart.</li> <li>• Experiences with probability that involve combinations occurs in grade five (e.g., How many different outfits can be made given three shirts and two pants?).</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Define probability as the measurement of chance that an event will happen.</li> <li>• List all possible outcomes for a single event (e.g., heads and tails are the two possible outcomes of flipping a coin). Limit the number of outcomes to 12 or fewer.</li> <li>• Describe the degree of likelihood of an outcome occurring using terms such as <i>impossible, unlikely, equally likely, likely, and certain</i>.</li> </ul> |

- 3.15 The student will**
- collect, organize, and represent data in pictographs or bar graphs; and**
  - read and interpret data represented in pictographs and bar graphs.**

| Understanding the Standard   | Essential Knowledge and Skills   |
|--|--|
| <ul style="list-style-type: none"> <li>Investigations involving data should occur frequently and relate to students' experiences, interests, and environment.</li> <li>Formulating questions for investigations is student-generated at this level. For example: What is the cafeteria lunch preferred by students in the class when four lunch menus are offered? If a new student enters our class tomorrow what color eyes will she likely have?</li> <li>The purpose of a graph is to represent data gathered to answer a question.</li> <li>A pictograph uses pictures or symbols to represent one or more objects.</li> <li>A key should be provided for the symbol in a pictograph when the symbol represents more than one piece of data (e.g.,  represents five people in a graph). The key is used in a graph to assist in the analysis of the displayed data. One-half of a symbol represents one-half of the value of the symbol being used, as indicated in the key.</li> <li>Students' prior knowledge and work with skip counting helps them to identify the number of pictures or symbols to be used in a pictograph.</li> <li>Definitions for the terms picture graph and pictographs vary. Pictographs are most often defined as a pictorial representation of numerical data. The focus of instruction should be placed on reading and using the key in analyzing the graph. There is no need for students to distinguish between a picture graph and a pictograph.</li> <li>Bar graphs are used to compare counts of different categories (categorical data). Using grid paper helps to increase accuracy in graphs. <ul style="list-style-type: none"> <li>A bar graph uses horizontal or vertical parallel bars to represent counts for categories. One bar is used for each category with the length of the bar representing the count for that category. There is space before, between, and after each of the bars.</li> <li>The axis displaying the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Grade three students should collect data that are recorded in increments of whole numbers, limited to multiples of 1, 2, 5, or 10.</li> <li>Each axis should be labeled, and the graph should be given a title.</li> </ul> </li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>Formulate questions to investigate. (a)</li> <li>Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. (a)</li> <li>Collect and organize data, using various forms of data collections (e.g., surveys, polls, questionnaires, scientific experiments, observations). (a)</li> <li>Represent data in a pictograph (limited to 16 or fewer data points for no more than four categories). (a)</li> <li>Represent data in a bar graph (limited to 16 or fewer data points for no more than four categories). (a) <ul style="list-style-type: none"> <li>Label each axis on a bar graph and give the bar graph a title. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. (a)</li> </ul> </li> <li>Analyze data represented in pictographs and bar graphs, orally and in writing. (b) <ul style="list-style-type: none"> <li>Read the information presented on a bar or pictograph (e.g., the title, the categories, the description of the two axes). (b)</li> </ul> </li> <li>Interpret information from pictographs and bar graphs, with up to 30 data points and up to eight categories, describe interpretation orally and by writing at least one sentence. (b) <ul style="list-style-type: none"> <li>Describe the categories of data and the data as a whole (e.g., data were collected on preferred ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad). (b)</li> </ul> </li> </ul> |


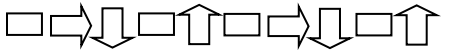
- 3.15 The student will**
- a) collect, organize, and represent data in pictographs or bar graphs; and**
  - b) read and interpret data represented in pictographs and bar graphs.**

| Understanding the Standard   | Essential Knowledge and Skills   |
|--|--|
| <ul style="list-style-type: none"> <li>• Statements representing an analysis and interpretation of the characteristics of the data in the graph should be written (e.g., similarities and differences, least and greatest, the categories, and total number of responses).</li> <li>• Statements should also express a prediction based on the analysis and interpretation of the characteristics of the data in the graph (e.g., The lunch room should serve pizza more often since that is the lunch students have liked the most.)</li> </ul> | <ul style="list-style-type: none"> <li>– Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). (b)</li> <li>– Select a correct interpretation of a graph from a set of interpretations, where one is correct and the remaining are incorrect. (b)</li> </ul> |



Students entering grades three through five have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

## 3.16 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

| Understanding the Standard   | Essential Knowledge and Skills  |
|--|---|
| <ul style="list-style-type: none"> <li>• Developing fluency and flexibility in identifying, describing, and extending patterns is fundamental to mathematics, particularly algebraic reasoning.</li> <li>• The use of materials to extend patterns permits experimentation or problem solving approaches that are almost impossible without them.</li> <li>• The simplest types of patterns are repeating patterns. In each case, students need to identify the core of the pattern and repeat it.</li> <li>• Growing patterns are more difficult for students to understand than repeating patterns because not only must they identify the core, they must also look for a generalization or relationship that will tell them how the pattern is changing from step to step. In many growing patterns the change can be described as an increase or decrease by a constant value. Students need experiences with growing patterns using objects, pictures, numbers, and tables.</li> <li>• In numeric patterns, students must determine the difference, called the <i>common difference</i>, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Students do not need to use the term <i>common difference</i> at this level.</li> <li>• Sample numeric patterns include: <ul style="list-style-type: none"> <li>– 6, 9, 12, 15, 18, ... (growing pattern);</li> <li>– 1, 2, 4, 7, 11, 16, ... (growing pattern);</li> <li>– 20, 18, 16, 14, ... (growing pattern); and</li> <li>– 1, 3, 5, 1, 3, 5, 1, 3, 5 ... (repeating pattern).</li> </ul> </li> <li>• Numeric patterns, at this level, will be limited to addition and subtraction of whole numbers.</li> <li>• In geometric figure patterns, students must often recognize transformations or changes of position in the plane of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line. Students at this level do not need to know the terms related to transformations of figures.</li> <li>• Sample geometric figure patterns include: <ul style="list-style-type: none"> <li>–  ... (growing pattern);</li> <li>–  ... (repeating pattern); and</li> </ul> </li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify and describe repeating and growing patterns using words, objects, pictures, numbers, and tables.</li> <li>• Identify a missing term in a pattern (e.g., 4, 6, □, 10, 12, 14).</li> <li>• Create repeating and growing patterns using objects, pictures, numbers, and tables.</li> <li>• Extend or identify missing parts in repeating and growing patterns using objects, pictures, numbers, and tables.</li> <li>• Solve problems that involve the application of input and output rules limited to addition and subtraction of whole numbers.</li> <li>• When given the rule, determine the missing values in a list or table. (Rules will be limited to addition and subtraction of whole numbers.)</li> </ul> |

## 3.16 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

| Understanding the Standard  | Essential Knowledge and Skills |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
|---|--------------------------------|--|-------|--------|---|---|---|---|---|---|------------------|--|-------|--------|---|---|---|---|---|---|--|
| <p>– □□★★□★★□□★★□★★... (repeating pattern).</p> <ul style="list-style-type: none"> <li>• Sample pattern transfers include:           <ul style="list-style-type: none"> <li>– 2, 5, 8, 11, 14, 17 has the same structure as 4, 7, 10, 13, 16, 19</li> <li>–  has the same structure as </li> </ul> </li> <li>• Input/output tables with a given rule provide direction for what to do to the input to get the output and can then be used to determine an unknown value. Applying rules to the input to find the output builds the foundation for functional thinking. Sample input/output tables that require applying the rule to the input to get the output:</li> </ul> <table border="1" data-bbox="407 732 625 899"> <thead> <tr> <th colspan="2">Rule: Add 4</th> </tr> <tr> <th>Input</th> <th>Output</th> </tr> </thead> <tbody> <tr> <td>4</td> <td>8</td> </tr> <tr> <td>5</td> <td>?</td> </tr> <tr> <td>8</td> <td>?</td> </tr> </tbody> </table> <table border="1" data-bbox="684 732 903 899"> <thead> <tr> <th colspan="2">Rule: Subtract 2</th> </tr> <tr> <th>Input</th> <th>Output</th> </tr> </thead> <tbody> <tr> <td>6</td> <td>?</td> </tr> <tr> <td>8</td> <td>6</td> </tr> <tr> <td>9</td> <td>?</td> </tr> </tbody> </table> | Rule: Add 4                    |  | Input | Output | 4 | 8 | 5 | ? | 8 | ? | Rule: Subtract 2 |  | Input | Output | 6 | ? | 8 | 6 | 9 | ? |  |
| Rule: Add 4   |                                |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| Input   | Output                         |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 4   | 8                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 5   | ?                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 8   | ?                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| Rule: Subtract 2  |                                |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| Input   | Output                         |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 6   | ?                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 8   | 6                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |
| 9   | ?                              |  |       |        |   |   |   |   |   |   |                  |  |       |        |   |   |   |   |   |   |  |



## 3.17 The student will create equations to represent equivalent mathematical relationships.

| Understanding the Standard   | Essential Knowledge and Skills   |
|--|--|
| <ul style="list-style-type: none"> <li>• Mathematical relationships can be expressed using equations (number sentences).</li> <li>• A number sentence is an equation with numbers (e.g., <math>6 + 3 = 9</math>; or <math>6 + 3 = 4 + 5</math>).</li> <li>• The equal symbol (<math>=</math>) means that the values on either side are equivalent (balanced).</li> <li>• The not equal (<math>\neq</math>) symbol means that the values on either side are not equivalent (not balanced).</li> <li>• An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., <math>5</math>, <math>4 + 3</math>, <math>8 - 2</math>, <math>2 \times 7</math>).</li> <li>• An equation is a mathematical sentence in which two expressions are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., <math>5 + 3 = 8</math>, <math>8 = 5 + 3</math> and <math>4 + 3 = 9 - 2</math>).</li> <li>• An equation can be represented using balance scales, with equal amounts on each side (e.g., <math>3 + 5 = 6 + 2</math>).</li> </ul> | <p><b>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</b></p> <ul style="list-style-type: none"> <li>• Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal (e.g., <math>256 - 13 = 220 + 23</math>; <math>143 + 17 = 140 + 20</math>; <math>457 + 100 \neq 557 + 100</math>).</li> <li>• Create equations to represent equivalent mathematical relationships (e.g., <math>4 \times 3 = 14 - 2</math>).</li> </ul> |