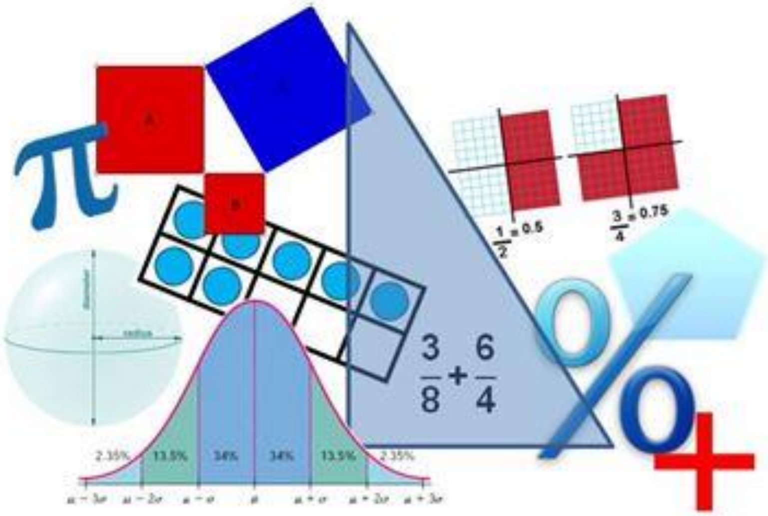
Mathematics

2016 Standards of Learning

Grade 8

Curriculum Framework



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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> “Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

8.1 The student will compare and order real numbers.

Understanding the Standard	Essential Knowledge and Skills
• Real numbers can be represented as integers, fractions (proper or improper), decimals, percents, numbers written in scientific notation, radicals, and π. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can have a positive or negative value. • The density property states that between any two real numbers lies another real number. For example, between 3 and 5 we can find 4; between 4.0 and 4.2 we can find 4.16; between 4.16 and 4.17 we can find 4.165; between 4.165 and 4.166 we can find 4.1655, etc. Thus, we can always find another number between two numbers. Students are not expected to know the term *density property* but the concept allows for a deeper understanding of the set of real numbers. • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π. Radicals may include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. • Use rational approximations (to the nearest hundredth) of irrational numbers to compare and order, locating values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.

8.2 The student will describe the relationships between the subsets of the real number system.

Understanding the Standard	Essential Knowledge and Skills
• The subsets of real numbers include natural numbers (counting numbers), whole numbers, integers, rational and irrational numbers. • Some numbers can belong to more than one subset of the real numbers (e.g., 4 is a natural number, a whole number, an integer, and a rational number). The attributes of one subset can be contained in whole or in part in another subset. The relationships between the subsets of the real number system can be illustrated using graphic organizers (that may include, but not be limited to, Venn diagrams), number lines, and other representations. • The set of natural numbers is the set of counting numbers {1, 2, 3, 4...}. • The set of whole numbers includes the set of all the natural numbers and zero {0, 1, 2, 3…}. • The set of integers includes the set of whole numbers and their opposites {…−2, −1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are $\sqrt{25}$, $\frac{1}{4}$, −2.3, 75%, and $4.\overline{59}$. • The set of irrational numbers is the set of all nonrepeating, nonterminating decimals. An irrational number cannot be written in fraction form (e.g., π, $\sqrt{2}$, 1.232332333…). • The real number system is comprised of all rational and irrational numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe and illustrate the relationships among the subsets of the real number system by using representations (graphic organizers, number lines, etc.). Subsets include rational numbers, irrational numbers, integers, whole numbers, and natural numbers. • Classify a given number as a member of a particular subset or subsets of the real number system, and explain why. • Describe each subset of the set of real numbers and include examples and non-examples. • Recognize that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

8.3 The student will

a) estimate and determine the two consecutive integers between which a square root lies; and

b) determine both the positive and negative square roots of a given perfect square.

Understanding the Standard	Essential Knowledge and Skills
• A perfect square is a whole number whose square root is an integer. • The square root of a given number is any number which, when multiplied times itself, equals the given number. • Both the positive and negative roots of whole numbers, except zero, can be determined. The square root of zero is zero. The value is neither positive nor negative. Zero (a whole number) is a perfect square. • The positive and negative square root of any whole number other than a perfect square lies between two consecutive integers (e.g.,$\sqrt{57}$ lies between 7 and 8 since 7^2 = 49 and 8^2 = 64; $-\sqrt{11}$ lies between -4 and -3 since $(-4)^2$ = 16 and $(-3)^2$ = 9). • The symbol $\sqrt{}$ may be used to represent a positive (principal) root and -$\sqrt{}$ may be used to represent a negative root. • The square root of a whole number that is not a perfect square is an irrational number (e.g., $\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a fraction $\frac{a}{b}$ where *b* does not equal 0. • Square root symbols may be used to represent solutions to equations of the form $x^2 = p$. Examples may include: – If x^2 = 36, then *x* is $\sqrt{36}$ = 6 or $-\sqrt{36}$ =−6. – If x^2 = 5, then *x* is $\sqrt{5}$ or $-\sqrt{5}$. • Students can use grid paper and estimation to determine what is needed to build a perfect square. The square root of a positive number is usually defined as the side length of a square with the area equal to the given number. If it is not a perfect square, the area provides a means for estimation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate and identify the two consecutive integers between which the positive or negative square root of a given number lies. Numbers are limited to natural numbers from 1 to 400. (a) • Determine the positive or negative square root of a given perfect square from 1 to 400. (b)

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

8.4 The student will solve practical problems involving consumer applications.

Understanding the Standard	Essential Knowledge and Skills
• Rational numbers may be expressed as whole numbers, integers, fractions, percents, and numbers written in scientific notation. • Practical problems may include, but are not limited to, those related to economics, sports, science, social science, transportation, and health. Some examples include problems involving the amount of a pay check per month, commissions, fees, the discount price on a product, temperature, simple interest, sales tax and installment buying. • A percent is a ratio with a denominator of 100. • Reconciling an account is a process used to verify that two sets of records (usually the balances of two accounts) are in agreement. Reconciliation is used to ensure that the balance of an account matches the actual amount of money deposited and/or withdrawn from the account. • A discount is a percent of the original price. The discount price is the original price minus the discount. • Simple interest (I) for a number of years is determined by finding the product of the principal (p), the annual rate of interest (r), and the number of years (t) of the loan or investment using the formula $I = prt$. • The total value of an investment is equal to the sum of the original investment and the interest earned. • The total cost of a loan is equal to the sum of the original cost and the interest paid. • Percent increase and percent decrease are both percents of change measuring the percent a quantity increases or decreases.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems involving consumer applications by using proportional reasoning and computation procedures for rational numbers. • Reconcile an account balance given a statement with five or fewer transactions. • Compute a discount or markup and the resulting sale price for one discount or markup. • Compute the sales tax or tip and resulting total. • Compute the simple interest and new balance earned in an investment or on a loan given the principal amount, interest rate, and time period in years. • Compute the percent increase or decrease found in a practical situation.

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one).

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

8.5 The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.

Understanding the Standard	Essential Knowledge and Skills
• Vertical angles are a pair of nonadjacent angles formed by two intersecting lines. Vertical angles are congruent and share a common vertex. • Complementary angles are any two angles such that the sum of their measures is 90°. • Supplementary angles are any two angles such that the sum of their measures is 180°. • Complementary and supplementary angles may or may not be adjacent. • Adjacent angles are any two non-overlapping angles that share a common ray and a common vertex.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the relationship between pairs of angles that are vertical, adjacent, supplementary, and complementary. • Use the relationships among supplementary, complementary, vertical, and adjacent angles to solve problems, including practical problems, involving the measure of unknown angles.

8.6 The student will
a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and
b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

Understanding the Standard	Essential Knowledge and Skills
• A polyhedron is a solid figure whose faces are all polygons. • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. • Surface area of a solid figure is the sum of the areas of the surfaces of the figure. • Volume is the amount a container holds. • A rectangular prism is a polyhedron that has a congruent pair of parallel rectangular bases and four faces that are rectangles. A rectangular prism has eight vertices and twelve edges. In this course, prisms are limited to right prisms with bases that are rectangles. • The surface area of a rectangular prism is the sum of the areas of the faces and bases, found by using the formula $S.A. = 2lw + 2lh + 2wh$. All six faces are rectangles. • The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism or by using the formula $V = lwh$. • A cube is a rectangular prism with six congruent, square faces. All edges are the same length. A cube has eight vertices and twelve edges. • A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this grade level, cones are limited to right circular cones. • The surface area of a right circular cone is found by using the formula, $S.A. = \pi r^2 + \pi rl$, where l represents the slant height of the cone. The area of the base of a circular cone is πr^2. • The volume of a cone is found by using $V = \frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base. • A square-based pyramid is a polyhedron with a square base and four faces that are triangles with a common vertex (apex) above the base. In this grade level, pyramids are limited to right regular pyramids with a square base. • The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between situations that are applications of surface area and those that are applications of volume. (a) • Determine the surface area of cones and square-based pyramids by using concrete objects, nets, diagrams and formulas. (a) • Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas. (a) • Solve practical problems involving volume and surface area of cones and square-based pyramids. (a) • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 2, 3, or 4. (b) • Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2. (b)

8.6 The student will

a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

Understanding the Standard	Essential Knowledge and Skills
• The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base, found by using the formula $S.A. = \frac{1}{2}lp + B$ where l is the slant height, p is the perimeter of the base and B is the area of the base. • The volume of a pyramid is found by using the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. • The volume of prisms can be found by determining the area of the base and multiplying that by the height. • The formula for determining the volume of cones and cylinders are similar. For cones, you are determining $\frac{1}{3}$ of the volume of the cylinder with the same size base and height. The volume of a cone is found by using $V = \frac{1}{3}\pi r^2 h$. The volume of a cylinder is the area of the base of the cylinder multiplied by the height, found by using the formula, $V = \pi r^2 h$, where h is the height and πr^2 is the area of the base. • The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator. • When the measurement of one attribute of a rectangular prism is changed through multiplication or division the volume increases by the same factor by which the attribute increased. For example, if a prism has a volume of $2 \cdot 3 \cdot 4$, the volume is 24 cubic units. However, if one of the attributes is doubled, the volume doubles. That is, $2 \cdot 3 \cdot 8$, the volume is 48 cubic units or 24 doubled. • When one attribute of a rectangular prism is changed through multiplication or division, the surface area is affected differently than the volume. The formula for surface area of a rectangular prism is $2(lw) + 2(lh) + 2(wh)$ when the width is doubled then four faces are affected. For example, a rectangular prism with length = 7 in., width = 4 in., and height = 3 in. would have a surface area of $2(7 \cdot 4) + 2(7 \cdot 3) + 2(4 \cdot 3)$ or 122 square inches. If the height is doubled to 6 inches then the surface area would be found by $2(7 \cdot 4) + 2(7 \cdot 6) + 2(4 \cdot 6)$ or 188 square inches.	

8.7 The student will

a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane; and

b) identify practical applications of transformations.

Understanding the Standard	Essential Knowledge and Skills
• Translations and reflections maintain congruence between the preimage and image but change location. Dilations by a scale factor other than 1 produce an image that is not congruent to the preimage but is similar. Reflections change the orientation of the image. • A transformation of a figure, called preimage, changes the size, shape, and/or position of the figure to a new figure, called the image. • A transformation of preimage point *A* can be denoted as the image *A′* (read as "*A* prime"). • A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. Each point on the image is the same distance from the line of reflection as the corresponding point in the preimage. • A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A dilation is a transformation in which an image is formed by enlarging or reducing the preimage proportionally by a scale factor from the center of dilation (limited to the origin in grade eight). A dilation of a figure and the original figure are similar. The center of dilation may or may not be on the preimage. • The result of first translating and then reflecting over the *x*- or *y*-axis may not result in the same transformation of reflecting over the *x*- or *y*-axis and then translating. • Practical applications may include, but are not limited to, the following: – A reflection of a boat in water shows an image of the boat flipped upside down with the water line being the line of reflection; – A translation of a figure on a wallpaper pattern shows the same figure slid the same distance in the same direction; and – A dilation of a model airplane is the production model of the airplane.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a preimage in the coordinate plane, identify the coordinate of the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been reflected over the *x*- or *y*-axis. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been dilated. Scale factors are limited to $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. (a) • Given a preimage in the coordinate plane, identify the coordinates of the image of a polygon that has been translated and reflected over the *x*-or *y*-axis, or reflected over the *x*- or *y*-axis and then translated. (a) • Sketch the image of a polygon that has been translated vertically, horizontally, or a combination of both. (a) • Sketch the image of a polygon that has been reflected over the *x*- or *y*-axis. (a) • Sketch the image of a dilation of a right triangle or a rectangle limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3, or 4. The center of the dilation will be the origin. (a) • Sketch the image of a polygon that has been translated and reflected over the *x*- or *y*-axis, or reflected over the *x*- or *y*-axis and then translated. (a)

8.7 The student will

a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane; and

b) identify practical applications of transformations.

Understanding the Standard	Essential Knowledge and Skills
	• Identify the type of translation in a given example. (a, b) • Identify practical applications of transformations including, but not limited to, tiling, fabric, wallpaper designs, art, and scale drawings. (b)

8.8 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

Understanding the Standard	Essential Knowledge and Skills
• A three-dimensional object can be represented as a two-dimensional model with views of the object from different perspectives. • Three-dimensional models of geometric solids can be used to understand perspective and provide tactile experiences in determining two-dimensional perspectives. • Three-dimensional models of geometric solids can be represented on isometric paper. • The top view is a mirror image of the bottom view.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct three-dimensional models, given the top or bottom, side, and front views. • Identify three-dimensional models given a two-dimensional perspective. • Identify the two-dimensional perspective from the top or bottom, side, and front view, given a three-dimensional model.

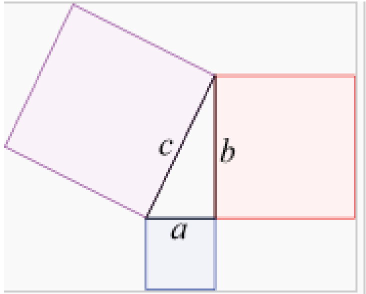
8.9 The student will

a) verify the Pythagorean Theorem; and

b) apply the Pythagorean Theorem.

Understanding the Standard

- The Pythagorean Theorem is essential for solving problems involving right triangles.
- The relationship between the sides and angles of right triangles are useful in many applied fields.
- In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the legs. This relationship is known as the Pythagorean Theorem: $a^2 + b^2 = c^2$.



- The Pythagorean Theorem is used to determine the measure of any one of the three sides of a right triangle if the measures of the other two sides are known.
- The converse of the Pythagorean Theorem states that if the square of the length of the hypotenuse equals the sum of the squares of the legs in a triangle, then the triangle is a right triangle. This can be used to determine whether a triangle is a right triangle given the measures of its three sides.
- Whole number triples that are the measures of the sides of right triangles, such as (3, 4, 5), (6, 8, 10), (9, 12, 15), and (5, 12, 13), are commonly known as Pythagorean triples.
- The hypotenuse of a right triangle is the side opposite the right angle.
- The hypotenuse of a right triangle is always the longest side of the right triangle.
- The legs of a right triangle form the right angle.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Verify the Pythagorean Theorem, using diagrams, concrete materials, and measurement. (a)
- Determine whether a triangle is a right triangle given the measures of its three sides. (b)
- Determine the measure of a side of a right triangle, given the measures of the other two sides. (b)
- Solve practical problems involving right triangles by using the Pythagorean Theorem. (b)

8.10 The student will solve area and perimeter problems, including practical problems, involving composite plane figures.

Understanding the Standard	Essential Knowledge and Skills
• A plane figure is any two-dimensional shape that can be drawn in a plane. • A polygon is a closed plane figure composed of at least three line segments that do not cross. • The perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. • The area of a composite figure can be found by subdividing the figure into triangles, rectangles, squares, trapezoids, parallelograms, circles, and semicircles, calculating their areas, and combining the areas together by addition and/or subtraction based upon the given composite figure. • The area of a rectangle is computed by multiplying the lengths of two adjacent sides ($A = lw$). • The area of a triangle is computed by multiplying the measure of its base by the measure of its height and dividing the product by 2 or multiplying by $\frac{1}{2}$ ($A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$). • The area of a parallelogram is computed by multiplying the measure of its base by the measure of its height ($A = bh$). • The area of a trapezoid is computed by taking the average of the measures of the two bases and multiplying this average by the height ($A = \frac{1}{2}h(b_1 + b_2)$). • The area of a circle is computed by multiplying pi times the radius squared ($A = \pi r^2$). • The circumference of a circle is found by multiplying pi by the diameter or multiplying pi by 2 times the radius ($C = \pi d$ or $C = 2\pi r$). • The area of a semicircle is half the area of a circle with the same diameter or radius.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Determine the area of subdivisions and combine to determine the area of the composite plane figure. • Subdivide a plane figure into triangles, rectangles, squares, trapezoids, parallelograms, and semicircles. Use the attributes of the subdivisions to determine the perimeter of the composite plane figure. • Apply perimeter, circumference, and area formulas to solve practical problems involving composite plane figures.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

8.11 The student will

a) compare and contrast the probability of independent and dependent events; and

b) determine probabilities for independent and dependent events.

Understanding the Standard	Essential Knowledge and Skills
• A simple event is one event (e.g., pulling one sock out of a drawer and examining the probability of getting one color). • If all outcomes of an event are equally likely, the theoretical probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space. • The probability of an event occurring can be represented as a ratio or the equivalent fraction, decimal, or percent. • The probability of an event occurring is a ratio between 0 and 1. A probability of zero means the event will never occur. A probability of one means the event will always occur. • Two events are either dependent or independent. • If the outcome of one event does not influence the occurrence of the other event, they are called independent. If two events are independent, then the probability of the second event does not change regardless of whether the first occurs. For example, the first roll of a number cube does not influence the second roll of the number cube. Other examples of independent events are, but not limited to: flipping two coins; spinning a spinner and rolling a number cube; flipping a coin and selecting a card; and choosing a card from a deck, replacing the card and selecting again. • The probability of two independent events is found by using the following formula: $P(A \text{ and } B) = P(A) \cdot P(B)$ − Example: When rolling a six-sided number cube and flipping a coin, simultaneously, what is the probability of rolling a 3 on the cube and getting a heads on the coin? $P(3 \text{ and heads}) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$ • If the outcome of one event has an impact on the outcome of the other event, the events are called dependent. If events are dependent then the second event is considered only if the first event has already occurred. For example, if you choose a blue card from a set of nine different colored cards that has a total of four blue cards and you do not place that blue card back in the set before selecting a second card, the chance of selecting a blue card the second time is diminished because there are now only three blue cards remaining in the set. Other examples of dependent events	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether two events are independent or dependent. (a) • Compare and contrast the probability of independent and dependent events. (a) • Determine the probability of two independent events. (b) • Determine the probability of two dependent events. (b)

8.11 The student will
a) compare and contrast the probability of independent and dependent events; and
b) determine probabilities for independent and dependent events.

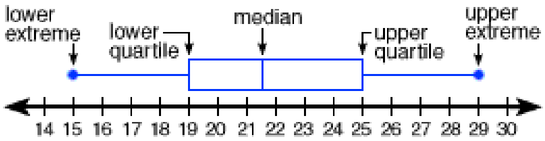
Understanding the Standard	Essential Knowledge and Skills
include, but are not limited to: choosing two marbles from a bag but not replacing the first after selecting it; determining the probability that it will snow and that school will be cancelled. • The probability of two dependent events is found by using the following formula: *P(A and B) = P(A)·P(B after A)* − Example: You have a bag holding a blue ball, a red ball, and a yellow ball. What is the probability of picking a blue ball out of the bag on the first pick then *without* replacing the blue ball in the bag, picking a red ball on the second pick? *P*(blue *and* red) = *P*(blue)·*P*(red *after* blue) = $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$	

8.12 The student will

a) represent numerical data in boxplots;
b) make observations and inferences about data represented in boxplots; and
c) compare and analyze two data sets using boxplots.

Understanding the Standard

- A boxplot (box-and-whisker plot) is a convenient and informative way to represent single-variable (univariate) data.
- Boxplots are effective at giving an overall impression of the shape, center, and spread of the data. It does not show a distribution in as much detail as a stem and leaf plot or a histogram.
- A boxplot will allow you to quickly analyze a set of data by identifying key statistical measures (median and range) and major concentrations of data.
- A boxplot uses a rectangle to represent the middle half of a set of data and lines (whiskers) at both ends to represent the remainder of the data. The median is marked by a vertical line inside the rectangle.
- The five critical points in a boxplot, commonly referred to as the five-number summary, are lower extreme (minimum), lower quartile, median, upper quartile, and upper extreme (maximum). Each of these points represents the bounds for the four quartiles. In the example below, the lower extreme is 15, the lower quartile is 19, the median is 21.5, the upper quartile is 25, and the upper extreme is 29.



- The range is the difference between the upper extreme and the lower extreme. The interquartile range (IQR) is the difference between the upper quartile and the lower quartile. Using the example above, the range is 14 or 29−15. The interquartile range is 6 or 25–19.
- When there are an odd number of data values in a set of data, the median will not be considered when calculating the lower and upper quartiles.
  - Example: Calculate the median, lower quartile, and upper quartile for the following data values:
    3 5 6 7 8 9 11 13 13
    Median: 8; Lower Quartile: 5.5; Upper Quartile: 12

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

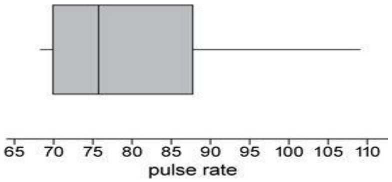
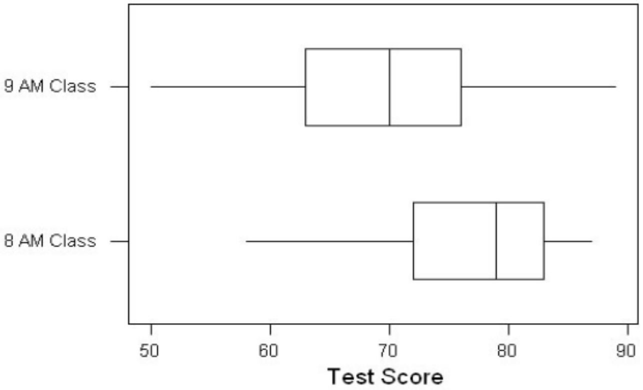
- Collect and display a numeric data set of no more than 20 items, using boxplots. (a)
- Make observations and inferences about data represented in a boxplot. (b)
- Given a data set represented in a boxplot, identify and describe the lower extreme (minimum), upper extreme (maximum), median, upper quartile, lower quartile, range, and interquartile range. (b)
- Compare and analyze two data sets represented in boxplots. (c)

8.12 The student will

a) represent numerical data in boxplots;

b) make observations and inferences about data represented in boxplots; and

c) compare and analyze two data sets using boxplots.

Understanding the Standard	Essential Knowledge and Skills
• In the pulse rate example, shown below, many students incorrectly interpret that longer sections contain more data and shorter ones contain less. It is important to remember that roughly the same amount of data is in each section. The numbers in the left whisker (lowest of the data) are spread less widely than those in the right whisker.  • Boxplots are useful when comparing information about two data sets. This example compares the test scores for a college class offered at two different times.  Using these boxplots, comparisons could be made about the two sets of data, such as comparing the median score of each class or the Interquartile Range (IQR) of each class.	

8.13 The student will

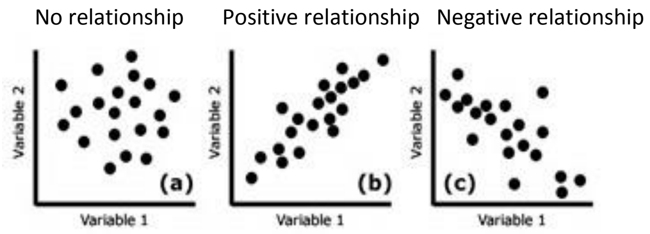
a) represent data in scatterplots;

b) make observations about data represented in scatterplots; and

c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

Understanding the Standard

- A scatterplot illustrates the relationship between two sets of numerical data represented by two variables (bivariate data). A scatterplot consists of points on the coordinate plane. The coordinates of the point represent the measures of the two attributes of the point.
- In a scatterplot, each point may represent an independent and dependent variable. The independent variable is graphed on the horizontal axis and the dependent is graphed on the vertical axis.
- Scatterplots can be used to predict linear trends and estimate a line of best fit.
- A line of best fit helps in making interpretations and predictions about the situation modeled in the data set. Lines and curves of best fit are explored more in Algebra I to make interpretations and predictions.
- A scatterplot can suggest various kinds of linear relationships between variables. For example, weight and height, where weight would be on *y*-axis and height would be on the *x*-axis. Linear relationships may be positive (rising) or negative (falling). If the pattern of points slopes from lower left to upper right, it indicates a positive linear relationship between the variables being studied. If the pattern of points slopes from upper left to lower right, it indicates a negative linear relationship.
  - For example: The following scatterplots illustrate how patterns in data values may indicate linear relationships.



Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, organize, and represent a data set of no more than 20 items using scatterplots. (a)
- Make observations about a set of data points in a scatterplot as having a positive linear relationship, a negative linear relationship, or no relationship. (b)
- Estimate the line of best fit with a drawing for data represented in a scatterplot. (c)

8.13 The student will

a) represent data in scatterplots;

b) make observations about data represented in scatterplots; and

c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

Understanding the Standard	Essential Knowledge and Skills
• A linear relationship between variables does not necessarily imply causation. For example, as the temperature at the beach increases, the sales at an ice cream store increase. If data were collected for these two variables, a positive linear relationship would exist, however, there is no causal relationship between the variables (i.e., the temperature outside does not cause ice cream sales to increase, but there is a relationship between the two). • The relationship between variables is not always linear, and may be modeled by other types of functions that are studied in high school and college level mathematics.	

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.

8.14 The student will

a) evaluate an algebraic expression for given replacement values of the variables; and

b) simplify algebraic expressions in one variable.

Understanding the Standard	Essential Knowledge and Skills
• An expression is a representation of a quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, 5x, 140 - 38.2, -18 · 21, (5 + 2x) · 4). An expression cannot be solved. • A numerical expression contains only numbers, the operations symbols, and grouping symbols. • Expressions are simplified using the order of operations. • Simplifying an algebraic expression means to write the expression as a more compact and equivalent expression. This usually involves combining like terms. • Like terms are terms that have the same variables and exponents. The coefficients do not need to match (e.g., 12*x* and -5*x*; 45 and $-5\frac{2}{3}$; 9*y*, -51*y* and $\frac{4}{9}y$.) • Like terms may be added or subtracted using the distributive and other properties. For example, − $2(x - \frac{1}{2}) + 5x = 2x - 1 + 5x = 2x + 5x - 1 = 7x - 1$ − $w + w - 2w = (1 + 1)\, w - 2w = 2w - 2w = (2 - 2)\, w = 0$ $w = 0$ • The order of operations is as follows: − First, complete all operations within grouping symbols*. If there are grouping symbols within other grouping symbols, do the innermost operation first. − Second, evaluate all exponential expressions. − Third, multiply and/or divide in order from left to right. − Fourth, add and/or subtract in order from left to right. * Parentheses ( ), brackets [ ], braces {}, absolute value | | (i.e., $\|3(-5+2)\| - 7$), and the division bar (i.e., $\frac{3+4}{5+6}$) should be treated as grouping symbols. • Properties of real numbers can be used to express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for the given replacement values of the variables. Exponents are limited to whole numbers and bases are limited to integers. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression. (a) • Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. (a) • Simplify algebraic expressions in one variable. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational. (b)

8.14 **The student will**
a) evaluate an algebraic expression for given replacement values of the variables; and
b) simplify algebraic expressions in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Associative property of addition: $(a + b) + c = a + (b + c)$. – Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. – Subtraction and division are neither commutative nor associative. – Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $a \cdot (b - c) = a \cdot b - a \cdot c$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Identity property of addition (additive identity property): $a + 0 = a$ *and* $0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ *and* $1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$]. – Inverse property of addition (additive inverse property): $a + (-a) = 0$ *and* $(-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ *and* $\frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0$ *and* $0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality.	

8.14 The student will

a) evaluate an algebraic expression for given replacement values of the variables; and

b) simplify algebraic expressions in one variable.

Understanding the Standard	Essential Knowledge and Skills
• A power of a number represents repeated multiplication of the number. For example, $(–5)^4$ means (–5) · (–5) · (–5) · (–5). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 4 is the exponent. The product is 625. Notice that the base appears inside the grouping symbols. The meaning changes with the removal of the grouping symbols. For example, $–5^4$ means 5 · 5 · 5 · 5 negated which results in a product of −625. The expression $– (5)^4$ means to take the opposite of 5 · 5 · 5 · 5 which is −625. Students should be exposed to all three representations. • An algebraic expression is an expression that contains variables and numbers. • Algebraic expressions are evaluated by substituting numbers for variables and applying the order of operations to simplify the resulting numeric expression.	

8.15 The student will

a) determine whether a given relation is a function; and

b) determine the domain and range of a function.

Understanding the Standard

- A relation is any set of ordered pairs. For each first member, there may be many second members.
- A function is a relation between a set of inputs, called the domain, and a set of outputs, called the range, with the property that each input is related to exactly one output.
- As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. In the "not a function" example, the input value "1" has two different output values, 5 and −3, assigned to it, so the example is not a function.

function

x	*y*
2	3
1	5
0	3
-1	-3

not a function

x	*y*
2	3
1	5
0	4
1	-3

- As a set of ordered pairs, a function has a unique or different *y*-value assigned to each *x*-value. For example, the set of ordered pairs, {(1, 2), (2, 4), (3, 2), (4, 8)} is a function. This set of ordered pairs, {(1, 2), (2, 4), (3, 2), (2, 3)}, is not a function because the *x*-value of "2" has two different *y*-values.
- As a graph of discrete points, a relation is a function when, for any value of *x*, a vertical line passes through no more than one point on the graph.
- Some relations are functions; all functions are relations.
- Graphs of functions can be discrete or continuous.
- In a discrete function graph there are separate, distinct points. You would not use a line to connect these points on a graph. The points between the plotted points have no meaning and cannot be interpreted. For example, the number of pets per household represents a discrete function because you cannot have a fraction of a pet.
- Functions may be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs. (a)
- Identify the domain and range of a function represented as a set of ordered pairs, a table, or a graph of discrete points. (b)

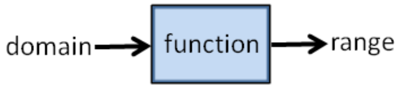
8.15 The student will

a) determine whether a given relation is a function; and

b) determine the domain and range of a function.

Understanding the Standard

- A discussion about determining whether a continuous graph of a relation is a function using the vertical line test may occur in grade eight, but will be explored further in Algebra I.



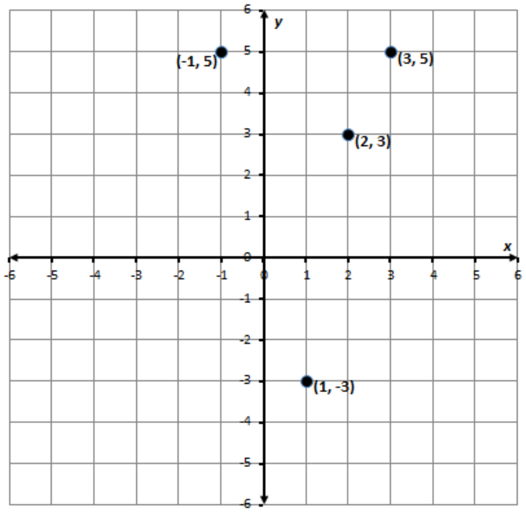
- The domain is the set of all the input values for the independent variable or *x*-values (first number in an ordered pair).
- The range is the set of all the output values for the dependent variable or *y*-values (second number in an ordered pair
- If a function is comprised of a discrete set of ordered pairs, then the domain is the set of all the *x*-coordinates, and the range is the set of all the *y*-coordinates. These sets of values can be determined given different representations of the function.
  - Example: The domain of a function is {−1, 1, 2, 3} and the range is {−3, 3, 5}. The following are representations of this function:
    - The function represented as a table of values:

x	y
-1	5
1	-3
2	3
3	5

    - The function represented as a set of ordered pairs: {(-1, 5), (1, -3), (2, 3), (3, 5)}

Essential Knowledge and Skills

8.15 **The student will**
a) determine whether a given relation is a function; and
b) determine the domain and range of a function.

Understanding the Standard	Essential Knowledge and Skills
○ The function represented as a graph on a coordinate plane: 	

8.16 The student will

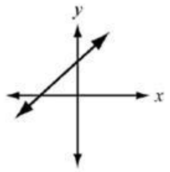
a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in *y* = *mx* + *b* form; and
e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

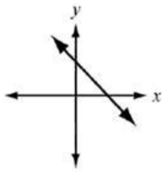
- A linear function is an equation in two variables whose graph is a straight line, a type of continuous function.
- A linear function represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same.
- Slope (m) represents the rate of change in a linear function or the “steepness” of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change.

$$\text{slope} = \frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$$

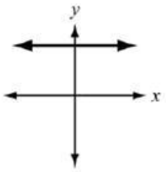
- A line is increasing if it rises from left to right. The slope is positive (i.e., $m > 0$).
- A line is decreasing if it falls from left to right. The slope is negative (i.e., $m < 0$).
- A horizontal line has zero slope (i.e., $m = 0$).



A line with a ***positive slope*** slants up to the right.



A line with a ***negative slope*** slants down to the right.



A line with a ***slope of 0*** is horizontal.

- A discussion about lines with undefined slope (vertical lines) should occur with students in grade eight mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize and describe a line with a slope that is positive, negative, or zero (0). (a)
- Given a table of values for a linear function, identify the slope and *y*-intercept. The table will include the coordinate of the *y*-intercept. (b)
- Given a linear function in the form *y* = *mx* + *b*, identify the slope and *y*-intercept. (b)
- Given the graph of a linear function, identify the slope and *y*-intercept. The value of the *y*-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers. (b)
- Identify the dependent and independent variable, given a practical situation modeled by a linear function. (c)
- Given the equation of a linear function in the form *y* = *mx* + *b*, graph the function. The value of the *y*-intercept will be limited to integers. (d)
- Write the equation of a linear function in the form *y* = *mx* + *b* given values for the slope, *m,* and the *y*-intercept or given a practical situation in which the slope, *m*, and *y*-intercept are described verbally.(e)

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;

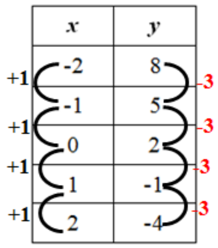
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in *y* = *mx* + *b* form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- A linear function can be written in the form $y = mx + b$, where m represents the slope or rate of change in y compared to x, and b represents the y-intercept of the graph of the linear function. The y-intercept is the point at which the graph of the function intersects the y-axis and may be given as a single value, b, or as the location of a point (0, b).
  - Example: Given the equation of the linear function $y = -3x + 2$, the slope is −3 or $\frac{-3}{1}$ and the y-intercept is 2 or (0, 2).
  - Example: The table of values represents a linear function.
    In the table, the point (0, 2) represents the y-intercept. The slope is determined by observing the change in each y-value compared to the corresponding change in the x-value.



x	*y*
-2	8
-1	5
0	2
1	-1
2	-4

$$\text{slope} = m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{-3}{+1} = -3$$

- The slope, m, and y-intercept of a linear function can be determined given the graph of the function.

Essential Knowledge and Skills

- Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e).

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;

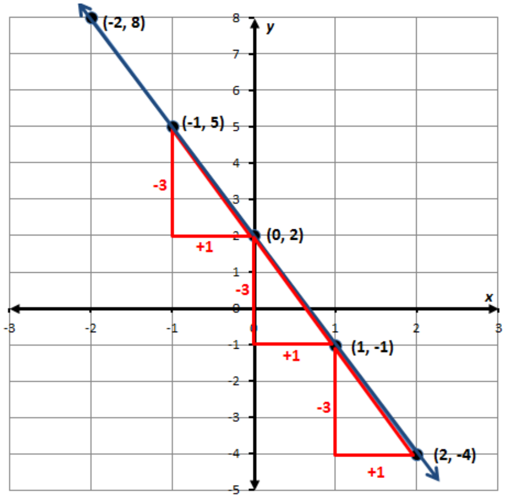
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in *y* = *mx* + *b* form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Example: Given the graph of the linear function, determine the slope and *y*-intercept.



Given the graph of a linear function, the *y*-intercept is found by determining where the line intersects the *y*-axis. The *y*-intercept would be 2 or located at the point (0, 2). The slope can be found by determining the change in each *y*-value compared to the change in each *x*-value. Here, we could use slope triangles to help visualize this:

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{-3}{+1} = -3$

- Graphing a linear function given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to

Essential Knowledge and Skills

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c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in *y* = *mx* + *b* form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills

determine points on the line.

- Example: Graph the linear function whose equation is $y = 5x - 1$.
  In order to graph the linear function, we can create a table of values by substituting arbitrary values for *x* to determining coordinating values for *y*:

x	$5x - 1$	y
-1	5(-1) - 1	-6
0	5(0) - 1	-1
1	5(1) - 1	4
2	5(2) - 1	9

The values can then be plotted as points on a graph.

Knowing the equation of a linear function written in $y = mx + b$ provides information about the slope and *y*-intercept of the function. If the equation is $y = 5x - 1$, then the slope, *m*, of the line is 5 or $\frac{5}{1}$ and the *y*-intercept is −1 and can be located at the point (0, −1). We can graph the line by first plotting the *y*-intercept. We also know,

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+5}{+1}$

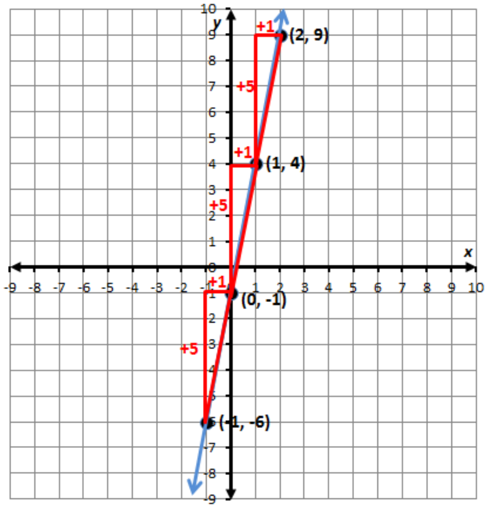
Other points can be plotted on the graph using the relationship between the *y* and *x* values.

Slope triangles can be used to help locate the other points as shown in the graph below:

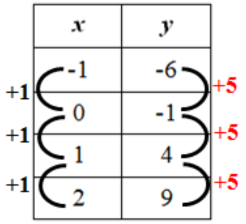
8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard



- A table of values can be used in conjunction with using slope triangles to verify the graph of a linear function. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.



x	y
-1	-6
0	-1
1	4
2	9

Essential Knowledge and Skills

8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in $y = mx + b$ form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in $y = mx + b$ form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard

- The axes of a coordinate plane are generally labeled x and y; however, any letters may be used that are appropriate for the function.
- A function has values that represent the input (x) and values that represent the output (y). The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.
- Below is a table of values for finding the approximate circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.

  - The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.
  - The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.
- In a graph of a continuous function every point in the domain can be interpreted. Therefore, it is possible to connect the points on the graph with a continuous line because every point on the line answers the original question being asked.
- The context of a problem may determine whether it is appropriate for ordered pairs representing a linear relationship to be connected by a straight line. If the independent variable (x) represents a discrete quantity (e.g., number of people, number of tickets, etc.) then it is not appropriate to connect the ordered pairs with a straight line when graphing. If the independent variable (x)

Essential Knowledge and Skills

8.16 The student will

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b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in $y = mx + b$ form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in $y = mx + b$ form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
represents a continuous quantity (e.g., amount of time, temperature, etc.), then it is appropriate to connect the ordered pairs with a straight line when graphing. – Example: The function $y = 7x$ represents the cost in dollars (y) for x tickets to an event. The domain of this function would be discrete and would be represented by discrete points on a graph. Not all values for x could be represented and connecting the points would not be appropriate. – Example: The function $y = -2.5x + 20$ represents the number of gallons of water (y) remaining in a 20-gallon tank being drained for x number of minutes. The domain in this function would be continuous. There would be an x-value representing any point in time until the tank is drained so connecting the points to form a straight line would be appropriate (Note: the context of the problem limits the values that x can represent to positive values, since time cannot be negative.). • Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations. • The equation $y = mx + b$ defines a linear function whose graph (solution) is a straight line. The equation of a linear function can be determined given the slope, m, and the y-intercept, b. Verbal descriptions of practical situations that can be modeled by a linear function can also be represented using an equation. – Example: Write the equation of a linear function whose slope is $\frac{3}{4}$ and y-intercept is −4, or located at the point (0, −4). The equation of this line can be found by substituting the values for the slope, $m = \frac{3}{4}$, and the y-intercept, $b = -4$, into the general form of a linear function $y = mx + b$. Thus, the equation would be $y = \frac{3}{4}x - 4$.	

8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard	Essential Knowledge and Skills
− Example: John charges a $30 flat fee to trouble shoot a personal watercraft that is not working properly and $50 per hour needed for any repairs. Write a linear function that represents the total cost, *y* of a personal watercraft repair, based on the number of hours, *x*, needed to repair it. Assume that there is no additional charge for parts. In this practical situation, the *y*-intercept, *b*, would be $30, to represent the initial flat fee to trouble shoot the watercraft. The slope, *m*, would be $50, since that would represent the rate per hour. The equation to represent this situation would be $y = 50x + 30$. • A proportional relationship between two variables can be represented by a linear function $y = mx$ that passes through the point (0, 0) and thus has a *y*-intercept of 0. The variable *y* results from *x* being multiplied by *m,* the rate of change or slope. • The linear function $y = x + b$ represents a linear function that is a non-proportional additive relationship. The variable *y* results from the value *b* being added to *x.* In this linear relationship, there is a y-intercept of *b,* and the constant rate of change or slope would be 1. In a linear function with a slope other than 1, there is a coefficient in front of the *x* term, which represents the constant rate of change, or slope. • Proportional relationships and additive relationships between two quantities are special cases of linear functions that are discussed in grade seven mathematics.	

8.17 The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• A multistep equation may include, but not be limited to equations such as the following: $2x + 1 = \frac{-x}{4}$; $-3(2x + 7) = \frac{1}{2}x$; $2x + 7 - 5x = 27$; $-5x - (x + 3) = -12$. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., $\frac{3}{4}$, $5x$, 140 − 38.2, 18 · 21, $5 + x$.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an "equal sign (=)". An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression "a number multiplied by five" could be represented by the variable expression "$n \cdot 5$" or "$5n$". • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x - 3$). • A verbal sentence is a complete word statement (e.g., "The sum of two consecutive integers is thirty-five." could be represented by "$n + (n + 1) = 35$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x + 3 = -4x + 1$). • In an equation, the "equal sign (=)" indicates that the value of the expression on the left is equivalent to the value of the expression on the right. • Like terms are terms that have the same variables and exponents. The coefficients do not need to match (e.g., $12x$ and $-5x$; 45 and $-5\frac{2}{3}$; $9y$, $-51y$ and $\frac{4}{9}y$.) • Like terms may be added or subtracted using the distributive and other properties. For example, – $4.6y - 5y = (-4.6 - 5)y = -9.6y$ – $w + w - 2w = (1 + 1)w - 2w = 2w - 2w = (2 - 2)w = 0 \cdot w = 0$ • Real-world problems can be interpreted, represented, and solved using linear equations in one variable.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and solve multistep linear equations in one variable with the variable on one or both sides of the equation (up to four steps) using a variety of concrete materials and pictorial representations. • Apply properties of real numbers and properties of equality to solve multistep linear equations in one variable (up to four steps). Coefficients and numeric terms will be rational. Equations may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a multistep linear equation. • Confirm algebraic solutions to linear equations in one variable.

8.17 The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.

Understanding the Standard

- Properties of real numbers and properties of equality can be used to solve equations, justify solutions and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard):
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Associative property of addition: $(a + b) + c = a + (b + c)$.
  - Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
  - Subtraction and division are neither commutative nor associative.
  - Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c \ and$ $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
  - Identity property of addition (additive identity property): $a + 0 = a \ and \ 0 + a = a$.
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \ and \ 1 \cdot a = a$.
  - Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$).
  - Inverse property of addition (additive inverse property): $a + (-a) = 0 \ and \ (-a) + a = 0$.
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \ and \ \frac{1}{a} \cdot a = 1$.
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0 \ and \ 0 \cdot a = 0$.
  - Division by zero is not a possible mathematical operation. It is undefined.

Essential Knowledge and Skills

8.17 The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of equality: If $a = b$, then $a + c = b + c$. – Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b \ and \ c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	

8.18 The student will solve multistep linear inequalities in one variable with the variable on one or both sides of the inequality symbol, including practical problems, and graph the solution on a number line.

Understanding the Standard

- A multistep inequality may include, but not be limited to inequalities such as the following: $2x + 1 > \frac{-x}{4}$; $-3(2x + 7) \le \frac{1}{2}x$; $2x + 7 - 5x < 27$; $-5x - (x + 3) > -12$.
- When both expressions of an inequality are multiplied or divided by a negative number, the inequality sign reverses.
- A solution to an inequality is the value or set of values that can be substituted to make the inequality true.
- In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solutions is $x > -7$. This means that x can be any number greater than −7. A few solutions might be −6.5, −3, 0, 4, 25, etc.)
- Real-world problems can be modeled and solved using linear inequalities.
- The properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Associative property of addition: $(a + b) + c = a + (b + c)$.
  - Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
  - Subtraction and division are neither commutative nor associative.
  - Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c\ and$ $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.
  - Identity property of addition (additive identity property): $a + 0 = a\ and\ 0 + a = a$.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve.
- Graph solutions to multistep linear inequalities on a number line.
- Write verbal expressions and sentences as algebraic expressions and inequalities.
- Write algebraic expressions and inequalities as verbal expressions and sentences.
- Solve practical problems that require the solution of a multistep linear inequality in one variable.
- Identify a numerical value(s) that is part of the solution set of a given inequality.

8.18 **The student will solve multistep linear inequalities in one variable with the variable on one or both sides of the inequality symbol, including practical problems, and graph the solution on a number line.**

Understanding the Standard	**Essential Knowledge and Skills**
– Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a\ and\ 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0\ and\ (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1\ and\ \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0\ and\ 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$. – Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$. – Multiplication property of inequality: If $a < b\ and\ c > 0$, then $a \cdot c < b \cdot c$; if $a > b\ and\ c > 0$, then $a \cdot c > b \cdot c$. – Multiplication property of inequality (multiplication by a negative number): If $a < b\ and\ c < 0$, then $a \cdot c > b \cdot c$; if $a > b\ and\ c < 0$, then $a \cdot c < b \cdot c$. – Division property of inequality: If $a < b\ and\ c > 0$, then $\frac{a}{c} < \frac{b}{c}$; if $a > b\ and\ c > 0$, then $\frac{a}{c} > \frac{b}{c}$. – Division property of inequality (division by a negative number): If $a < b\ and\ c < 0$, then $\frac{a}{c} > \frac{b}{c}$; if $a > b\ and\ c < 0$, then $\frac{a}{c} < \frac{b}{c}$.	