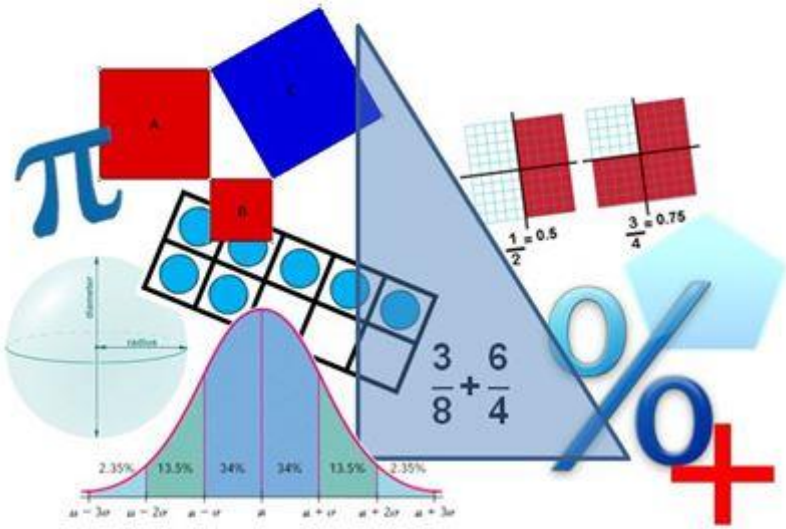
Mathematics

2016 Standards of Learning

Kindergarten
Curriculum Framework



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Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning Curriculum Framework*.

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Students in kindergarten through grade two have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

K.1 The student will

a) tell how many are in a given set of 20 or fewer objects by counting orally; and

b) read, write, and represent numbers from 0 through 20.

Understanding the Standard	Essential Knowledge and Skills
• There are three developmental levels of counting: – rote sequence; – one-to-one correspondence; and – the cardinality of numbers. • Counting involves two separate skills: verbalizing the list (rote sequence counting) of standard number words in order ("one, two, three, …") and connecting this sequence with the objects in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. Objects may be presented in random order or arranged for easy counting. • When counting objects, students should: – Say the number names in standard order; – Count one item for each number word (one-to-one correspondence); – Understand that the number of objects is the same regardless of their arrangement or the order in which they were counted (conservation of number); – Understand that the last number names the total amount of objects counted (cardinality); and – Understand that each successive number name refers to a quantity that is one larger. • Cardinality is knowing how many are in a set by recognizing that the last counting word tells the total number in a set. • After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, "How many are there?" Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them. • Kinesthetic involvement (e.g., tracing the numerals, using tactile materials, such as sand, sandpaper, carpeting, or finger paint) facilitates the writing of numerals. • If a set is empty, it has zero objects or elements. Zero is both a number and a digit. It is used as a placeholder in our number system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count orally to tell how many are in a given set containing 20 or fewer concrete objects, using one-to-one correspondence, and identify the corresponding numeral. (a) • Read, write, and represent numbers from 0-20 to include: – Construct a set of objects that corresponds to a given numeral, including an empty set; – Read and write the numerals from 0 through 20; – Identify written numerals from 0 through 20 represented in random order; – Identify the numeral that corresponds to the total number of objects in a given set of 20 or fewer concrete objects; and – Write a numeral that corresponds to a set of 20 or fewer concrete objects. (b)

K.1 The student will

a) tell how many are in a given set of 20 or fewer objects by counting orally; and

b) read, write, and represent numbers from 0 through 20.

Understanding the Standard	Essential Knowledge and Skills
• Symbolic reversals in numeral writing are common for this age and should not be mistaken for lack of understanding. • Describing a teen number as a ten and some more, will help students name how many are in a set of 13-19 objects. This also lays a foundation for place value.	

K.2 The student, given no more than three sets, each set containing 10 or fewer concrete objects, will

a) compare and describe one set as having more, fewer, or the same number of objects as the other set(s); and

b) compare and order sets from least to greatest and greatest to least.

Understanding the Standard	Essential Knowledge and Skills
• A set is a collection of objects. • Sets can be compared by matching, lining up the objects, visually estimating magnitude, recognizing quantities without counting (subitizing), or counting the number of objects in each set. • Comparing sets is an extension of conservation of number (e.g., 5 is 5 whether it is 5 marbles or 5 basketballs even though 5 basketballs take up more space). When comparing objects, the set can be arranged differently while still containing the same number (e.g., 5 marbles in a cup is the same as 5 marbles on the floor). • Comparing objects is an extension of cardinality. Cardinality is knowing how many are in a set by recognizing that the last counting word tells the total number in a set. • Students are generally familiar with the concept of *more*, but may have had little experience with the term *fewer*. It is important to use the terms together to build an understanding of their relationship. For example, when asking which group has more, follow with which group has fewer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and describe no more than three sets of 10 or fewer objects, using the terms *more, fewer,* and *the same*. (a) • Given a set of objects, construct a second set which has more, fewer, or the same number of objects. (a) • Compare and order three or fewer sets, each set containing 10 or fewer concrete objects, from least to greatest and greatest to least. (b)

K.3 The student will

a) count forward orally by ones from 0 to 100;

b) count backward orally by ones when given any number between 1 and 10;

c) identify the number after, without counting, when given any number between 0 and 100 and identify the number before, without counting, when given any number between 1 and 10; and

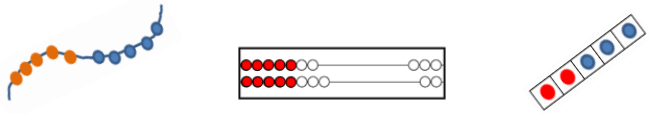
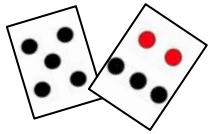
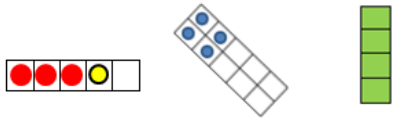
d) count forward by tens to determine the total number of objects to 100.

Understanding the Standard	Essential Knowledge and Skills
• Counting skills are essential components of the development of number ideas; however, they are only one of the indicators of the understanding of numbers. • Counting forward by rote, supported by visuals such as the hundred chart or number path, advances the child's development of sequencing. • The natural numbers are 1, 2, 3, 4…. The whole numbers are 0, 1, 2, 3, 4…. Students should count the whole numbers 0, 1, 2, 3, 4…. • A number path is a counting model where each number is represented within a square and the squares can be clearly counted. Example of a Number Path 1 2 3 4 5 6 7 8 9 10 • A number line is a length model where each number represents its length from zero. When young children use a number line as a counting tool, they often confuse what should be counted (the numbers or the spaces between the numbers). A number path is more appropriate for students at this age. • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 10, 9, 8,… through …3, 2, 1, 0. • Counting forward and backward leads to the development of counting on and counting back. • Connecting rote counting to the counting of collections is necessary for students to understand the meaning of a number. • Identifying the number after and/or the number before any given numbers demonstrates an understanding of number relationships as opposed to a memorized sequence of numbers. • Providing experiences in counting beyond 100 will help students who often struggle with going over the century mark.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count forward orally by ones from 0 to 100. (a) • Count backward orally by ones when given any number between 1 and 10. (b) • Identify the number after, without counting, when given any number between 0 and 100. (c) • Identify the number before, without counting, when given any number between 1 and 10. (c) • Count forward orally by tens, starting at 0, to determine the total number of objects up to 100. (d)

K.4 The student will

a) recognize and describe with fluency part-whole relationships for numbers up to 5; and

b) investigate and describe part-whole relationships for numbers up to 10.

Understanding the Standard	Essential Knowledge and Skills
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved. • Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures. • Parts of 5 and 10 should be represented in a variety of ways, such as five frames, ten frames, strings of beads, arrangements of tiles or tooth picks, dot cards, or beaded number frames.  • Dot patterns should be presented in both regular and irregular arrangements. This will help students to understand that numbers are made up of parts, and will later assist them in combining parts as well as counting on.  • Numbers can be composed and decomposed using part-part-whole relationships (e.g., 4 can be decomposed as 3 and 1, 2 and 2, 4 and 0).  • Quickly recognizing and naming the number of objects in a small group without counting is called subitizing. The size of the group a student can subitize is dependent upon the arrangement of the dots or objects. At this age, students should subitize regular arrangements up to 5. • When students are able to combine or separate groups to create a number, they are building a foundation for addition and subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe with fluency part-whole relationships for numbers up to 5 in a variety of configurations. (a) • Investigate and describe part-whole relationships for numbers up to 10 using a variety of configurations. (b)

K.4 The student will

a) recognize and describe with fluency part-whole relationships for numbers up to 5; and

b) investigate and describe part-whole relationships for numbers up to 10.

Understanding the Standard	Essential Knowledge and Skills
• Benchmarks of 5 and 10 are essential in building place value knowledge through the understanding of decomposition of the numbers of 5 and 10. • Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships. • Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or losing track of the logic of the strategy being used. • Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers they do not know.	

K.5 The student will investigate fractions by representing and solving practical problems involving equal sharing with two sharers.

Understanding the Standard	Essential Knowledge and Skills
• Practical situations with fractions should involve real-life problems in which students themselves determine how to subdivide a whole into equal parts, testing those parts to be sure they are equal, and using those parts to re-create the whole. • Fractions can have different meanings: part-whole, division, measurement, ratio, and operator. The focus of this grade level is to develop the idea of equal sharing (division) and part-whole relationships. Fraction notation will be introduced in grade two. • Young children understand equal sharing problems intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples: – Two children sharing six sandwiches – Two children sharing one sandwich – Two children sharing four brownies For two children sharing one sandwich, a child might say that each will get half of the sandwich. For two children sharing four brownies, a child might say they each will get half of the brownies, while another child might say they will get one of the two pieces. • Teachers should use vocabulary such as halves. Students may name the parts as halves but may also use language such as "one piece out of the two pieces" to describe half. Students at this level should not be expected to use fraction vocabulary or notation. Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development. • Students should be encouraged to create drawings or use concrete objects or other representations to solve problems. • Fraction models at this level should be able to be continuously divided (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed. • In each fraction model, the fractional parts must be equal shares of a whole. • Equal parts may be different shapes but maintain the same value (e.g., a sandwich could be cut in two equal pieces vertically, horizontally, or diagonally to represent halves). • The fraction name *half* tells the number of equal parts in the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Share a whole equally with two sharers, when given a practical situation. • Represent fair shares concretely or pictorially, when given a practical situation. • Describe shares as equal pieces or parts of the whole (e.g., halves), when given a practical situation.

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In kindergarten through grade two, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and subtraction within 20;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable; and
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects as well as problems that involve sharing equally, and initial work with fractions.

K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10, using concrete objects.

Understanding the Standard

- Students should experience a variety of problem types related to addition and subtraction.
- The problem types most appropriate for students at this level include:

KINDERGARTEN: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES	
Join (Result Unknown)	Sue had 4 pennies. Josh gave her 2 more. How many pennies does Sue have altogether?
Separate (Result Unknown)	Sue had 8 pennies. She gave 5 pennies to Josh. How many pennies does Sue have now?
Part-Part-Whole (Whole Unknown)	Josh has 4 red balloons and 3 blue balloons. How many balloons does he have?
Part-Part-Whole (Both Parts Unknown)	Josh has 5 balloons. Some of them are red and some of them are blue. How many balloons can be blue and how many can be red?

- Join problems involve the process of combining or joining sets or quantities. Separate problems can be viewed as a taking away or separating process. Part-part-whole problems involve two quantities that are combined into one whole but no physical action is required. Comparison problems that ask *how many more* or *how many fewer* should be reserved for grades one and two.
- Operation symbols (+, -) are introduced in grade one.
- Single-step refers to the least number of steps necessary to solve a problem.
- Number relationships help students develop strategies for addition and subtraction. These strategies include:
  - Instant recognition of the amount in a set of objects (subitize) that are arranged in a familiar pattern such as the dots on number cubes; and
  - One more than, one less than, two more than, two less than.
- Counting on from the larger set to determine the sum of the combined sets is one strategy for determining a sum.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model and solve various types of story and picture problems using 10 or fewer concrete objects. (Types of problems should include joining, separating, and part-part-whole scenarios.)

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in kindergarten through grade two encounter measurement in their daily lives, from their use of the calendar and science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight, capacity, time, temperature, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain a deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply several techniques, direct comparison, nonstandard units, and standard tools to determine measurements and to develop an understanding of the use of U.S. Customary units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on:

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three).
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the number of pennies equivalent to a nickel, a dime, and a quarter.

Understanding the Standard	Essential Knowledge and Skills
• Involvement in varied activities such as physically manipulating coins and making comparisons about their sizes, colors, and values are prerequisites to the skills of coin recognition and valuation. • Students need experiences counting collections of pennies. This can promote one-to-one correspondence, as a penny is worth one cent. • Students need experiences to develop the concept that a nickel has a value of five cents (which is the same as five pennies), that a dime has a value of 10 cents (which is the same as ten pennies), and a quarter has a value of 25 cents (which is the same as twenty-five pennies), even though each coin (nickel, dime, quarter) is only one object.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe the attributes (e.g., color, relative size) of a penny, nickel, dime, and quarter. • Identify a penny, nickel, dime, and quarter. • Identify the number of pennies equivalent to a nickel, a dime, and a quarter (i.e., a nickel has the same value as five pennies).

K.8 The student will investigate the passage of time by reading and interpreting a calendar.

Understanding the Standard	Essential Knowledge and Skills
• Practical situations are appropriate to develop a sense of the interval of time between events (e.g., club or team meetings occur every week on Monday, there is a week between meetings). • The calendar is a way to represent units of time (e.g., days, weeks, months, and a year). • Using a calendar develops the concept of a day as a 24-hour period rather than a period of time from sunrise to sunset.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Name the twelve months of the year. • Name the seven days in a week. • Determine the day before and after a given day (e.g., yesterday, today, tomorrow).

K.9 The student will compare two objects or events, using direct comparisons, according to one or more of the following attributes: length (longer, shorter), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter).

Understanding the Standard	Essential Knowledge and Skills
• Students need to identify the attribute that they are measuring (e.g., length, height, weight, temperature, volume) before they begin to measure. • Multiple hands-on experiences are needed to gain the ability to compare the attributes of objects. • Students develop conservation of measurement when they understand that the attributes do not change when the object is manipulated (e.g., a piece of string that is coiled maintains its length as it is straightened; the volume of water does not change when poured from a pitcher into a fish tank.) • Length is the distance between two points. • Height is the distance from the bottom or base of something to the top. • Weight is a measure of the heaviness of an object. • Temperature is the degree of hotness or coldness of an object or environment. • Volume is the measure of the capacity of a container. • Time is the measure of an event from its beginning to end. Students could compare the difference between the time spent sliding down the slide versus the time spent walking around the school building.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and describe lengths of two objects as longer or shorter, using direct comparison (e.g., the bus is longer than the car). • Compare and describe heights of two objects (as taller or shorter), using direct comparison. • Compare and describe weights of two objects (as heavier or lighter), using direct comparison. • Compare and describe temperatures of two objects or environment (as hotter or colder), using direct comparison. • Compare and describe volumes of two containers (as more or less), using direct comparison. • Compare and describe the amount of time spent on two events (as longer or shorter), using direct comparison.

K.10 The student will

a) identify and describe plane figures (circle, triangle, square, and rectangle);

b) compare the size (smaller, larger) and shape of plane figures (circle, triangle, square, and rectangle); and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

Understanding the Standard	Essential Knowledge and Skills
• An important part of the geometry strand in kindergarten through grade two is the naming and describing of figures. Children move from their own vocabulary and begin to incorporate conventional terminology as the teacher uses geometric terms. • Early experiences with comparing, sorting, combining, and subdividing figures assist students in analyzing the characteristics of plane figures. • Attribute blocks and tangrams are among the manipulatives that are particularly appropriate for sorting and comparing size and shape. • Students should be given opportunities to construct plane figures using multiple tools (e.g., clay, straws, paper, and scissors). • Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in their environment by the type of figures those things represent. • Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students are less likely to develop common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. • A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing (e.g., a square is a square regardless of its location in space; there is no plane figure called a diamond). • A plane figure is any closed, two-dimensional shape. • A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. The term *vertices* is the plural form of vertex. • A polygon is a closed plane figure composed of at least three line segments that do not cross. • A triangle is a polygon with three sides.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a circle, triangle, square, and rectangle. (a) • Describe the characteristics of triangles, squares, and rectangles, including number of sides and number of vertices. (a) • Describe a circle using terms such as *round* and *curved*. (a) • Compare and group plane figures (circle, triangle, square, and rectangle) according to their relative sizes (smaller, larger). (b) • Compare and group plane figures (circle, triangle, square, and rectangle) according to their shapes. (b) • Distinguish between examples and nonexamples of identified plane figures (circle, triangle, square, and rectangle). (b) • Identify pictorial representations of a circle, triangle, square, and rectangle, regardless of their position and orientation in space. (c) • Describe the location of one object relative to another, using the terms *above*, *below*, and *next to*. (c)

K.10 The student will

a) identify and describe plane figures (circle, triangle, square, and rectangle);

b) compare the size (smaller, larger) and shape of plane figures (circle, triangle, square, and rectangle); and

c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

Understanding the Standard	Essential Knowledge and Skills
• Children should have experiences with different types of triangles (e.g., equilateral, isosceles, scalene, right, acute, obtuse); however, at this level, they are not expected to name the various types. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a quadrilateral with four congruent (equal length) sides and four right angles. At this level, students might describe a square as a special rectangle with four sides of equal length. • Students at this level do not need to use the terms *polygon*, *quadrilateral*, or *congruent*. • A circle is the set of points in a plane that are the same distance from a point called the center. A circle is not a polygon because it does not have straight sides.	

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. In grade two, students experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

K.11 The student will

a) collect, organize, and represent data; and

b) read and interpret data in object graphs, picture graphs, and tables.

Understanding the Standard	Essential Knowledge and Skills
• Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is to inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites). • Methods for organizing data could include five or ten frames, surveys, checklists, or various methods of grouping concrete materials. • At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than four categories. • Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who brought their lunch and who will buy their lunch for any given day, a picture graph showing how students traveled to school – bus, car, walk). • When data are presented in an organized manner, students can interpret and discuss the results and implications of their investigation (e.g., identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same number of responses). • In the process of collecting data, students make decisions about what is relevant to their investigation (e.g., when collecting data on their classmates' favorite pets, deciding to limit the categories to common pets). • When students begin to collect data, they recognize the need to categorize, which helps develop the understanding of "things that go together." Categorical data are used when constructing picture graphs and bar graphs. • Different types of representations emphasize different things about the same data. • Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). • Picture graphs are graphs that use pictures to represent and compare information. At this level, each picture should represent one data point.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data on categories identified by the teacher and/or student (e.g., number of siblings, types/numbers of pets, types of flowers in the garden). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) • Represent data by arranging concrete objects into organized groups to form a simple object graph. (a) • Represent gathered data, using pictures to form a simple picture graph (e.g., a picture graph of the weather for a month). (a) • Represent gathered data in tables (vertically or horizontally). (a) • Answer questions related to the gathered data displayed in object graphs, picture graphs, and tables: – Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., five people are wearing sneakers); and – Interpret the data that represents numerical relationships, including categories with the greatest, the least, or the same. (b)

K.11 The student will

a) collect, organize, and represent data; and

b) read and interpret data in object graphs, picture graphs, and tables.

Understanding the Standard	Essential Knowledge and Skills
• Tables are an orderly arrangement of data in columns and rows in an essentially rectangular format. Tables may be used to display numerical relationships or to organize lists. • Students represent data to convey results of their investigations at a glance, using concrete objects, pictures, and numbers to give a "picture" of the organized data. • Graphs can be used to make connections between mathematics and science or social studies (e.g., types of plants found in the school yard, how students get to school). • Students should have experiences answering questions related to the analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses).	

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through grade two develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

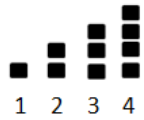
- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

K.12 The student will sort and classify objects according to one attribute.

Understanding the Standard	Essential Knowledge and Skills
• Objects can be sorted and classified in different ways. • To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). • General similarities and differences among objects are easily observed by children entering kindergarten, who are able to focus on any one attribute. The teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves having both different colors and different figures).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the attributes of an object (e.g., color, size, shape, thickness) • Sort objects into appropriate groups (categories) based on one attribute (e.g., size – large bears and small bears). • Classify sets of objects into groups (categories) of one attribute. • Label attributes of a set of objects that has been sorted. • Name multiple ways to sort a set of objects.

K.13 The student will identify, describe, extend, create, and transfer repeating patterns.

Understanding the Standard	Essential Knowledge and Skills
• Patterning is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning. • Opportunities to create, identify, describe, extend, and transfer repeating patterns are essential to the primary school experience and lay the foundation for thinking algebraically. • Patterning should include: – creating a given pattern, using objects, sounds, movements, and pictures; – recording a pattern with pictures or symbols; – transferring a pattern into a different representation (e.g., the pattern snap, snap, clap changed to a blue, blue, red pattern, or changed to an AAB repeating pattern); and – analyzing patterns in practical situations (e.g., calendar, seasons, days of the week). • The part of the pattern that repeats is called the core. • At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABCABCABC) as well as when the final repetition of the core is incomplete (e.g., ABCABCA... or Red, Blue, Green, Red, Blue, Green, Red, Blue…). • Examples of repeating patterns: – ABABABAB; – ABCABC; – ABBAABBA; – AABBAABBAABB; and – AABAAB. • Examples of growing patterns, introduced in grade one, include: –  – **10, 20, 30, 40, 50…** – ☆△☆☆△☆☆☆△	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe the core (the part of the sequence that repeats) found in repeating patterns of common objects, sounds, movements, and pictures. • Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern. • Create a repeating pattern. • Compare similarities and differences between patterns. • Transfer a repeating pattern from one representation to another.